

Unit 8 Review

$$A = \begin{pmatrix} 1 & 7 \\ 4 & 3 \\ \boxed{8} & -7 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -9 & 32 \\ -8 & 12 & 0 \end{pmatrix}$$

3×2 2×3

What is the order of A? B?

What is the order of AB ? BA ?

$$3 \times (2 \cdot 2) \times 3 = 3 \times 3$$

$$(2 \times 3) \cdot (3 \times 2) = 2 \times 2$$

What is A_{31} ? $\frac{11}{11} = 8$

Write the augmented matrix for the system and the matrix equation

$$x - 3y + z = 4$$

$$-x + 2y - 5z = 3$$

$$5x - 13y + 13z = 8$$

Write the system of equations from the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 5 & -13 & 13 & 8 \end{array} \right)$$

$$\begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix} + \begin{pmatrix} 3 & -5 \\ -2 & 7 \end{pmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 5 & 4 \\ -3 & 6 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -5 \\ -2 & 7 \\ -3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ -1 & 5 \\ -2 & 3 \end{pmatrix} = \begin{bmatrix} 0 \cdot 2 - 6 & 2 \cdot -10 + 9 \\ 0 + 3 - 8 & 4 - 15 + 12 \end{bmatrix}$$

$2 \times 3 \cdot 3 \times 2$

$$\begin{bmatrix} 2 & 7 & 4 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -5 & 1 \end{bmatrix}$$

$1 \times 3 \cdot 1 \times 3$

Now - to manipulate our matrix:

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Back

we use row operations

- interchange any 2 rows
- multiply all elements of a row by a nonzero real number
- add a multiple of one row to any other row

BACK

Our goal - is row echelon form (REF)

$$\begin{pmatrix} 1 & -1 & 2 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix} \quad \text{if there are any rows with all 0's they are at the bottom}$$

or better yet - reduced row echelon form (RREF)

$$\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Notation:

1. R_{ij} means exchange rows i and j
2. kR_i means multiply i th row by k
3. $kR_i + R_j$ means adding k times the i th row to the j th row

$$x - 3y + z = 4$$

$$-y - 4z = 7$$

$$5x - 13y + 13z = 8$$

How do you know if there are infinite solutions or no solutions?

What does the graph look like for infinite solutions?

$$\left[\begin{array}{ccc|c} 2 & 3 & 7 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 3 \end{array} \right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x + y + z = 3$$

$$2x + y + 4z = 8$$

$$x + 2y - z = 1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 1 \end{bmatrix}$$

$A \qquad B$

$$A^{-1} \cdot B = V$$

Singular Matrix

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$$\begin{aligned} x + 2y + z &= -1 \\ x - 3y + 2z &= 1 \\ 2x - 3y + z &= 5 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -3 & 2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$$

$(1/4, -3/4, -7/4)$

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ -2 & -4 & -2 & 2 \\ 2 & -3 & 1 & 5 \end{bmatrix} \xrightarrow{\begin{matrix} -R_1+R_2 \\ -2R_1+R_3 \end{matrix}} \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & -5 & -4 & 4 \\ 0 & -7 & -1 & 7 \end{bmatrix}$$

$$\xrightarrow{7R_2+R_3} \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/5 & -2/5 \\ 0 & 0 & -12/5 & 24/5 \end{bmatrix} \xrightarrow{\frac{5}{12}R_3} \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/5 & -2/5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} -\frac{1}{5}R_2 \\ -\frac{5}{12}R_3 \end{matrix}} \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/5 & -2/5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$-\frac{5}{12} \cdot \frac{21}{5} = -\frac{21}{12}$

$$\begin{bmatrix} 2 & 6 & 8 & 22 \\ 1 & 2 & 3 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 4 & 11 \\ 1 & 2 & 3 & 7 \end{bmatrix}$$

Review of Inverses

A matrix has an inverse if:

- It is Square
- The $\det(A) \neq 0$

If the $\det(A)=0$ then the matrix is SINGULAR

The determinant of a 2x2 matrix is $ad-bc$

Inverse of a 2x2 Matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\underline{ad-bc}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}^{-1} = \frac{1}{6-4} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$$

2x2 Matrices: BY HAND

$$3x - 2y = 0$$

$$-x + y = 5$$

Solve by substitution and elimination

$$2x - y = 10$$

$$3x + 2y = 1$$

Word Problems

A Sparrow flying with the wind from American Fork to Sandy takes 1.35 hours. Flying against the wind the sparrow takes 2.27 hours for the return trip. If the air distance between American Fork and Sandy is 45 miles and the sparrow speed and wind speed are constant, find the sparrow speed and the wind speed.

$$\frac{m}{h} \quad \begin{aligned} S + W &= \frac{45}{1.35} \\ S - W &= \frac{45}{2.27} \end{aligned}$$

You have tootsie rolls that sell for \$3.25 a lb and fruity tootsie rolls that sell for \$5.75 a pound. How much of each must you mix to get 100 pounds of a mixture that you can sell for \$4.00 per pound? Express the problem as a system of linear equations and solve using the method of your choice.

$$\begin{aligned} t + f &= 100 \\ \frac{3.25t + 5.75f}{100} &= 4.00 \\ 3.25t + 5.75f &= 400 \end{aligned}$$

