

## Unit 9 Review

9.1/9.3 Combinations, permutations and probability

$${}_n C_r = \frac{n!}{r!(n-r)!}$$

Committee

$${}_n P_r = \frac{n!}{(n-r)!}$$

Presidency

$$\text{Probability} = \frac{\text{\# of desired outcomes}}{\text{\# of possible outcomes}}$$

16. How many ways can 4 red flowers, 3 yellow flowers, and 6 purple flowers be planted in a row?

$$13! \quad 6,227,020,800$$

17. What is the probability of drawing 3 kings and 2 queens from a standard deck of 52 cards?

How many "words" can you make with the letters from: MISSISSIPPI

$$\frac{11!}{4!4!2!}$$

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$11!$

## 9.2 The Binomial Theorem

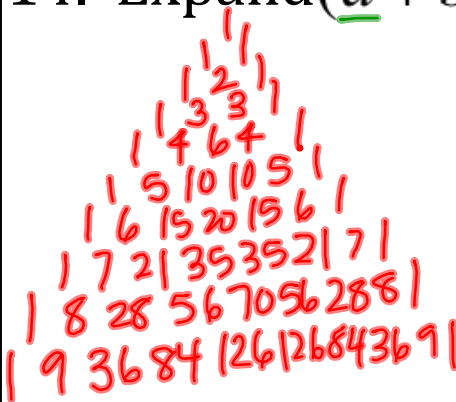
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (a+b)^n \longrightarrow \binom{n}{r} a^{n-r} b^r$$

=

 $nC_r$ 

can use calculator :)

14. Expand  $(a + b)^9$  using a method of your choice.



$$\begin{aligned}
 & \binom{9}{0}a^9 + \binom{9}{1}a^8b + \binom{9}{2}a^7b^2 + \binom{9}{3}a^6b^3 + \binom{9}{4}a^5b^4 \\
 & + \binom{9}{5}a^4b^5 + \binom{9}{6}a^3b^6 + \binom{9}{7}a^2b^7 + \binom{9}{8}ab^8 + \binom{9}{9}b^9 \\
 & a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 \\
 & + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9
 \end{aligned}$$

15. Find the  $x^8$  term in the expansion of  $(x - 2)^{12}$

$$\begin{aligned}
 \binom{12}{8} x^8 (-2)^4 &= 12C_8 = 495 (-2)^4 x^8 \\
 &= \boxed{7920 x^8}
 \end{aligned}$$

Note: A red box highlights the term  $(x-2)^{12}$  in the problem statement, and a red arrow points from the exponent 8 in  $x^8$  to the corresponding term in the expansion.

Expand  $(x - 2y)^4$  using the binomial theorem

## 9.4 Sequences

Arithmetic

$$a_n = a_1 + d(n-1)$$

 $n^{\text{th}}$  term of arithmetic

Geometric

$$a_n = a_1 \cdot r^{n-1}$$

 $n^{\text{th}}$  term

Explicit vs. Recursive

2, 4, 8, 16, 32, ...

$$b_1 = -1 \text{ and } b_{k+1} = b_k + 10 \quad \text{for } k \geq 1$$

Find an explicit formula for the sequence, find the 10th term

$$\boxed{-6}, -2, 2, 6, 10, \dots$$

$$d=4$$

$$a_n = -6 + 4(n-1)$$

$$a_n = -6 + 4n - 4$$

$$\boxed{a_n = 4n - 10}$$

$$a_{10} = 4(10) - 10$$

$$\boxed{a_{10} = 30}$$

Find an explicit formula for the sequence, find the 10th term

$$2, 6, 18, 54, \dots$$

$$\boxed{a_n = 2 \cdot (3)^{n-1}}$$

$$a_{10} = 2 \cdot 3^{10-1}$$



## 9.4 Series

$$\sum_{k=1}^n a_k$$

Sum of a finite  
arithmetic sequence

$$* S_n = \frac{n}{2}(a + a_n)$$

sum of finite  
geometric sequence

$$* S_n = \frac{a(1-r^n)}{1-r}$$

Sum of infinite  
geometric sequence

$|r| < 1$   
converges

$$S_n = \frac{a_1}{1-r}$$

$|r| \geq 1$   
diverges

$$\sum_{k=1}^{\infty}$$

$$\sum_{n=1}^5 n^2 - 1$$

$$(1^2-1) + (2^2-1) + (3^2-1) + (4^2-1) + (5^2-1) = \boxed{50}$$

$$\sum_{j=3}^5 j - j^2$$

$$\sum_{k=0}^4 \left(\frac{1}{2}\right)^k$$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{1(1-\frac{1}{2}^5)}{1-\frac{1}{2}}$$

$$\boxed{\frac{31}{16}}$$

$$\sum_{n=0}^5 (-1)^n n^3$$

$$\sum_{n=1}^{\infty} 4\left(\frac{2}{3}\right)^{n-1}$$

$|\frac{2}{3}| < 1$  converges

$$S = \frac{a}{1-r} = \frac{4}{1-\frac{2}{3}} = \boxed{12}$$

$$\sum_{n=0}^{\infty} 3\left(-\frac{1}{3}\right)^{n+1}$$

$$\sum_{k=0}^{\infty} 2\left(\frac{1}{5}\right)^k$$

✓

$$\sum_{k=1}^n \underline{2k+3}$$

$$(2(1)+3) + (2(2)+3) + \dots + (2(n)+3)$$

$$4, \frac{-4}{3}, \frac{4}{9}, \frac{-4}{27}, \dots, 4\left(\frac{-1}{3}\right)^{10}$$

9.7/9.8 Statistics

Stem and Leaf plots

$\downarrow$   
 $32.7$   
 $42.1$   
 $27.3$

27	3
28	
29	
30	
31	
32	7

Five number Summary  
 $\{min, Q_1, med, Q_3, max\}$

Standard Deviation

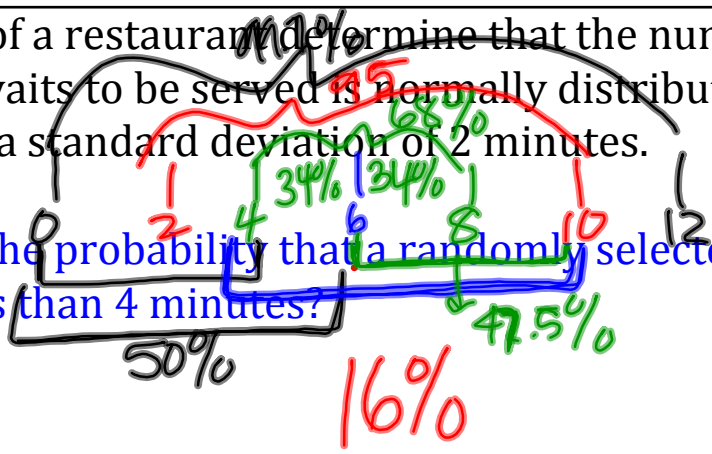
Calc.  $\sigma$

Variance  
 $\sigma^2$

68-95-99.7 Rule

$68\%$  within  $\pm 1\sigma$   
 $95\%$  within  $\pm 2\sigma$   
 $99.7\%$  within  $\pm 3\sigma$

The owners of a restaurant determine that the number of minutes that a customer waits to be served is normally distributed with a mean of 6 minutes and a standard deviation of 2 minutes.



11. What is the probability that a randomly selected customer will be served in less than 4 minutes?

12. During a survey, 500 customers are served. How many would you expect to be served in less than 8 minutes?

$$64\% \quad .64(500) = \boxed{420}$$

13. If 1000 customers are served, how many would you expect to wait between 4 minutes and 10 minutes?

$$34 + 47.5 = 81.5\% \quad (1000)(.815) = \boxed{815}$$