Unit 9 Review

9.1/9.3 Combinations, permutations and probability

\[ nC_r = \frac{n!}{r!(n-r)!} \]

\[ nP_r = \frac{n!}{(n-r)!} \]

Committee \hspace{1cm} Presidency

Probability = \( \frac{\text{#of desired outcomes}}{\text{#of possible outcomes}} \)
16. How many ways can 4 red flowers, 3 yellow flowers, and 6 purple flowers be planted in a row?

\[\frac{13!}{3! \cdot 2! \cdot 6!} = 4,227,020,800\]

17. What is the probability of drawing 3 kings and 2 queens from a standard deck of 52 cards?

How many "words" can you make with the letters from: MISSISSIPPI

\[\frac{11!}{4! \cdot 4! \cdot 2!}\]
16. How many ways can 4 red flowers, 3 yellow flowers, and 6 purple flowers be planted in a row?

\[ 13! \quad 6,227,020,800 \]

17. What is the probability of drawing 3 kings and 2 queens from a standard deck of 52 cards?

How many "words" can you make with the letters from: MISSISSIPPI

\[ \underline{\text{}} \]
9.2 The Binomial Theorem

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (a+b)^n \rightarrow \binom{n}{r} a^{n-r} b^r
\]

\(nC_r\) can use calculator.
14. Expand \((a + b)^9\) using a method of your choice.

\[
\begin{align*}
&11 \quad 11 \\
&10 \quad 45 \\
&9 \quad 36 \\
&8 \quad 28 \\
&7 \quad 21 \\
&6 \quad 15 \\
&5 \quad 10 \\
&4 \quad 6 \\
&3 \quad 1 \\
&2 \quad 1 \\
&1 \quad 1
\end{align*}
\]

\[
(9)a^9 + (36)a^7b^2 + (84)a^5b^4 + (72)a^3b^6 + (36)a^1b^8 + (9)b^{10}
\]

15. Find the \(x^8\) term in the expansion of \((x - 2)^{12}\).

\[
\binom{12}{8} x^8 (-2)^4 = 12 \cdot 1 = 495 \cdot (-2)^4 \cdot x^8
\]

\[
= 7920x^8
\]
Expand \( (x - 2y)^4 \) using the binomial theorem
9.4 Sequences

Arithmetic

\[ a_n = a + d(n-1) \]

Geometric

\[ a_n = a \cdot r^{n-1} \]

Explicit vs. Recursive

2, 4, 8, 16, 32, ...

\[ b_1 = -1 \quad \text{and} \quad b_{k+1} = b_k + 10 \quad \text{for} \quad k \geq 1 \]
Find an explicit formula for the sequence, find the 10th term

-6, -2, 2, 6, 10, ...

\[ a_1 = -6 \]
\[ a_n = a_{n-1} + 4 \]
\[ a_n = -6 + 4(n-1) \]
\[ a_n = 4n - 10 \]
\[ a_{10} = 4(10) - 10 \]
\[ a_{10} = 30 \]

Find an explicit formula for the sequence, find the 10th term

2, 6, 18, 54, ...

\[ a_1 = 2 \]
\[ r = 3 \]
\[ a_n = a_1 \cdot r^{n-1} \]
\[ a_n = 2 \cdot 3^{n-1} \]
\[ a_{10} = 2 \cdot 3^{10-1} \]
9.4 Series

\[ \sum_{k=1}^{n} a_k \]

Sum of a finite arithmetic sequence

\[ S_n = \frac{n}{2}(a + a_n) \]

Sum of finite geometric sequence

\[ S_n = \frac{a(1 - r^n)}{1 - r} \]

Sum of infinite geometric sequence

\[ |r| < 1 \]
\[ S_n = \frac{a_1}{1 - r} \]
\[ \text{converges} \]

\[ |r| \geq 1 \]
\[ \text{diverges} \]
\[
\sum_{n=1}^{5} n^2 - 1 + (1^2-1) + (2^2-1) + (3^2-1) + (4^2-1) + (5^2-1) = 50
\]

\[
\sum_{j=3}^{5} j - j^2
\]

\[
\sum_{k=0}^{4} \left( \frac{1}{2} \right)^k
\]

\[
5^n = a_1 (1 - r^n) = \frac{1 - (1 - \frac{1}{2})^{5}}{1 - r} = \frac{1 - \frac{1}{31/2}}{1 - r}
\]

\[
\sum_{n=0}^{5} (-1)^n n^3
\]
$$\sum_{n=1}^{\infty} 4\left(\frac{2}{3}\right)^{n-1}$$

\[|\frac{2}{3}| < 1 \text{ converges} \]

\[S = \frac{a}{1-r} = \frac{4}{1-\frac{2}{3}} + 12\]

$$\sum_{n=0}^{\infty} 3\left(-\frac{1}{3}\right)^{n+1}$$

$$\sum_{k=1}^{n} 2k + 3$$

\[= (2(1)+3) + (2(2)+3) + \ldots + (2n+3)\]
4, \frac{-4}{3}, \frac{4}{9}, \frac{-4}{27}, \ldots, 4 \left(\frac{-1}{3}\right)^{10}
9.7/9.8 Statistics

Stem and Leaf plots

Five number Summary
\[ \text{min}, q_1, med, q_3, \text{max} \]

Standard Deviation
Calc. \( \sigma x \)

Variance
\( \sigma^2 \)

68-95-99.7 Rule

68% within \( \pm 1 \sigma \)
96% within \( \pm 2 \sigma \)
99.7% within \( \pm 3 \sigma \)
The owners of a restaurant determine that the number of minutes that a customer waits to be served is normally distributed with a mean of 6 minutes and a standard deviation of 2 minutes.

11. What is the probability that a randomly selected customer will be served in less than 4 minutes?

12. During a survey, 500 customers are served. How many would you expect to be served in less than 8 minutes?

13. If 1000 customers are served, how many would you expect to wait between 4 minutes and 10 minutes?