Review #1
P. 2

Distance Formula: distance between points \(P(x_1, y_1)\) and \(Q(x_2, y_2)\) is

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Find the distance between \((1,5)\) and \((6,2)\)

\[
\sqrt{(6-1)^2 + (2-5)^2} = \sqrt{25 + 9} = \sqrt{34}
\]

Midpoint Formula: midpoint between points \(P(x_1, y_1)\) and \(Q(x_2, y_2)\) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Find the midpoint between \((1,5)\) and \((6,2)\)

\[
\left( \frac{1+6}{2}, \frac{5+2}{2} \right) = \left( \frac{7}{2}, \frac{7}{2} \right)
\]
P.3/P.4

General Form: \( Ax + By + C = 0 \)
Slope Intercept: \( y = mx + b \)
Point-Slope: \( y - y_1 = m(x - x_1) \)
Vertical Line: \( x = a \)
Horizontal Line: \( y = b \)

Parallel and Perpendicular
1. Two nonvertical lines are parallel iff their slopes are equal.

2. Two nonvertical lines are perpendicular iff their slopes are opposite reciprocals. Iff

\[ m_1 = -\frac{1}{m_2} \]
P.5

Quadratic Equations:
A Quadratic equation in $x$ is one that can be written in the form $a, b, c \in \mathbb{R}, a \neq 0$

$$ax^2 + bx + c = 0$$

Ways to solve Quadratic equations

Extracting Square Roots $(ax + b)^2 = c$

Example: $(2x - 1)^2 = 9$

$2x - 1 = \pm 3$

$x = \frac{1 \pm 3}{2}$

Completing the Square

Solve by the Quadratic Formula

FACTORIZING
P.6
Operations w/ Complex #'s

add/subtract: combine like terms (real w/real and imaginary w/imaginary)

Divide: factor or use conjugates to simplify (remember \(i\) is a radical and can't stay in the denominator)

**Complex Conjugates:** 2 complex #'s \(a + bi\) and \(a - bi\) are conjugates because when multiplied all imaginary parts are eliminated

\[(3 - 2i)(4 + 3i)\]
Absolute Value Inequalities

\[ \geq, > \quad \text{great"or"} \quad \text{"or" inequality} \]

\[ \leq, < \quad \text{less th"and"} \quad \text{"and" inequality} \]

To solve absolute value inequalities:
1. Isolate the absolute value on the left.
2. Decide if you have a "or" or "and" inequality
3. Write the inequalities.
4. Solve the inequalities.
5. Graph to find "and" intersection / "or union"
6. Check your answer in the original problem
7. Write the answer in interval notation.
Solving Quadratic Inequalities

1. Make right side of the inequality 0
2. Decide if you have an "or" or "and" inequality
3. Solve the quadratic equation
4. Graph the quadratic and decide which values are above or below x-axis
5. Write in interval notation

\[ x^2 - x - 12 > 0 \]
1.2

**y-intercepts:** where the graph crosses the y-axis and $x = 0$

**x-intercepts:** where the graph crosses the x-axis and $y = 0$

**Function:** when each domain value is paired with only one range value (no repeating x's)

- graphically: passes the vertical line test

**Domain & Range (card)**

**Domain:** x-values - input

- read x's from left to right (smallest to largest)

*some functions have domain restrictions - can't divide by zero

to find: set the den. = 0 and solve for x. These are the restrictions.

can't have a neg. # in a sq. root

to find: set the radicand $\geq 0$ and solve for x.

**Range:** y-values - output

- read y's from bottom to top (smallest to largest)
**Domain Restrictions:**

1. Exclude any value that makes the denominator = 0

2. Exclude values that lead to the $\sqrt{}$ of a negative number

3. Taking the Log of a negative number

\[
f(x) = 2x^{473} - 2x^4
\]

D:\((-\infty, \infty)\)

\[
f(x) = \sqrt{3-x}
\]

D:\((-\infty, 3]\)

\[
f(x) = \frac{1}{\sqrt{3-x}}
\]

D:\((-\infty, 3)\)
Asymptotes:

vertical (VA): caused by dividing by 0
the graph approaches $-\infty$ or $\infty$
on each side of the asymptote
to find the asymptote set den = 0 and solve

dend behavior: (horizontal (HA) or oblique (OA)):
to find the asymptote - compare the degrees of the num and den. if
- top heavy (OA):
- bottom heavy (HA): $y = 0$
- equal (HA): divide coefficients
- oblique: (more later)
Increasing, Decreasing and Constant

· as you move from left to right the y-values increase
· as you move from left to right the y-values decrease
· as you move from left to right the y-values do not change

This behavior is reported using interval notation for the x-values where the graph has a certain behavior.

Extrema

maximums
· relative (local)
· absolute (upper bound)

minimums
· relative (local)
· absolute (lower bound)
Odd/Even/Neither Symmetry (card title)

Odd \[ f(-x) = - f(x) \]
symmetry with respect to the origin

Even \[ f(-x) = f(x) \]
symmetry with respect to the y-axis

Neither
1.3

- $f(x) = x$
- $f(x) = x^3$
- $f(x) = |x|$
- $f(x) = \sin x$
- $f(x) = \sqrt{x}$
- $f(x) = \ln x$
- $f(x) = x^2$
- $f(x) = e^x$
- $f(x) = \frac{1}{x}$
- $f(x) = \cos x$
Piecewise Functions

certain pieces of the function have specific behavior

frequently: intervals (parts) of the domain are associated with different functions (related to continuity)

\[ f(x) = \begin{cases} 
    x + 1 & \text{if } x \leq 0 \\
    x & \text{if } x > 0 
\end{cases} \]
Composition of Functions - defined

\[(f \circ g)(x) = f(g(x))\]

Finding the domain of a composition

\[f(x) = x^2 - 1\]
\[g(x) = \sqrt{x}\]

\[(g \circ f)(x)\]

\[(f \circ g)(x)\]

1. What is the domain of the first function?
2. Find the domain restrictions of the new function
3. Put them together
1.5
Finding an Inverse Algebraically (card)
Steps:
1. replace f(x) or relation name w/ y if not in that form
2. switch the x & y in the eq. (just x & y not signs, coefficients, or exponents)
3. Solve for y.
4. replace y with relation name $^{-1}$ (f$^{-1}$ or g$^{-1}$)

\[ y = \frac{3x}{2x+7} \]
\[ x = \frac{3y}{2y+7} \]
\[ (2y+1)x = 3y \]
\[ 2xy + x = 3y \]
\[ -3y = -3y \]
\[ 2xy - 3y + 7x = 0 \]
\[ -7x = -7x \]
\[ 2xy - 3y = -7x \]
\[ y(2x-3) = -7x \]
\[ \frac{2x-3}{2x-3} \]
\[ y = \frac{-7x}{2x-3} \]
\[ y = \pm \Theta f(\pm \#(x \pm \Delta)) \pm \text{■} \]

- **±** if (-) reflection over x-axis (range Δ)
- **Θ** vertical expansion or compression (range Δ)
  - Θ > 1 expansion
  - Θ < 1 compression
- **±** if (-) reflection over y-axis (domain Δ)
- **#** horizontal expansion or compression (domain Δ)
  - 0 < # < 1 expansion
  - # > 1 compression
- **Δ** translation left or right (domain Δ)
  - (+) left (-) right
- **■** translation up or down (range Δ)
  - (+) up (-) down

x's lie

Domain changes
Range changes
2.1 Quadratic Equations:
Standard Form:
\[ y = ax^2 + bx + c \]
Graphing or Vertex Form:
\[ y = a(x - h)^2 + k \]
vertex: (h,k)
axis of symmetry \( x=h \)
End Behavior (polynomial)

2.3

End Behavior is determined by the degree of the polynomial and the coefficient of the leading term. The mathematical notation is written using limits.

\[
\lim_{x \to -\infty} f(x) = \text{left end}
\]

\[
\lim_{x \to \infty} f(x) = \text{right end}
\]

Odd Degree: the left & right ends go in opp. directions

(+) coeff. \hspace{2cm} (-) coeff.

\[
\lim_{x \to \infty} f(x) = \infty \hspace{2cm} \lim_{x \to -\infty} f(x) = -\infty
\]

\[
\lim_{x \to -\infty} f(x) = -\infty \hspace{2cm} \lim_{x \to \infty} f(x) = \infty
\]

Even Degree: both ends go in the same direction

(+) coeff. \hspace{2cm} (-) coeff.

both up \hspace{2cm} both down

\[
\lim_{x \to \infty} f(x) = \infty \hspace{2cm} \lim_{x \to -\infty} f(x) = -\infty
\]

\[
\lim_{x \to -\infty} f(x) = \infty \hspace{2cm} \lim_{x \to \infty} f(x) = -\infty
\]
The power of the factor determines the nature of the intersection at the point $x = a$. (This is referred to as the multiplicity.)

**Straight intersection:**

$(x - a)^1$ The power of the zero is 1.

**Tangent intersection:**

$(x - a)^{\text{even}}$ The power of the zero is even.

**Inflection intersection:** (like a slide through)

$(x - a)^{\text{odd}}$ The power of the zero is odd.