Review #1

P. 2

Distance Formula: distance between points $P(x_1, y_1)$ and $Q(x_2,y_2)$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

34

2 17

Find the distance between (1,5) and (6,2)

$$\sqrt{(6-1)^2 + (2-5)^2}$$

5^2

$25 + 9$ $\boxed{\sqrt{34}}$

Midpoint Formula: midpoint between points $P(x_1, y_1)$ and $Q(x_2,y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the midpoint between (1,5) and (6,2)

$$\left(\frac{1+6}{2}, \frac{5+2}{2}\right) \quad \left(\frac{7}{2}, \frac{7}{2}\right)$$

P.3/P.4

General Form: $Ax + By + C = 0$

Slope Intercept: $y = mx + b$

Point-Slope: $y - y_1 = m(x - x_1)$

Vertical Line: $x = a$

Horizontal Line: $y = b$

Parallel and Perpendicular

1. Two nonvertical lines are parallel iff their slopes are equal.

2. Two nonvertical lines are perpendicular iff their slopes are opposite reciprocals. Iff

$$m_1 = -\frac{1}{m_2}$$

December 08, 2011

P.5

Quadratic Equations:

A Quadratic equation in x is one that can be written in the form $a,b,c \ni \mathbb{R}, a \neq 0$

$$ax^2 + bx + c = 0$$

Ways to solve Quadratic equations

Extracting Square Roots $(ax+b)^2 = c$

Example: $\sqrt{(2x-1)^2} = \sqrt{9}$ $2x-1 = \pm 3$ $x = \frac{1 \pm 3}{2}$

$\boxed{x = 2, -1}$

Completing the Square

Solve by the Quadratic Formula

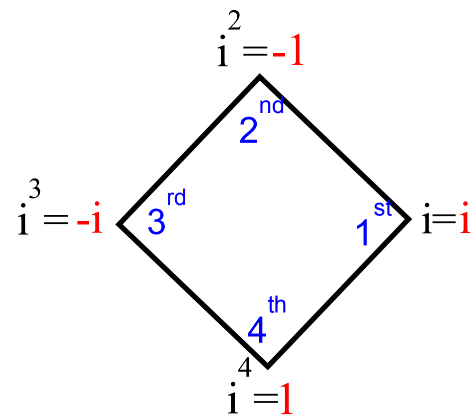
FACTORING

P.6

Operations w/ Complex #'s

add/subtract: combine like terms (real w/real and imaginary w/imaginary)

Divide: factor or use conjugates to simplify (remember *i* is a radical and can't stay in the denominator)



Complex Conjugates: 2 complex #'s a + bi and a - bi are conjugates because when multiplied all imaginary parts are eliminated

$i^{40} i^3 = (1) i^3 = \boxed{-i}$

$\boxed{i^{43}}$

$(3 - 2i)(4 + 3i)$

$12 + 9i - 8i - 6i^2$

$\cancel{12}\ 18 + i$

$\frac{(2+4i)}{(2+4i)} \cdot \frac{3i}{2-4i}$

$\frac{6i + 12i^2}{4 \cancel{-8i} \cancel{+8i} - 16i^2}$

$\frac{-12+6i}{20} = \frac{-6+3i}{10}$

P.7

Absolute Value Inequalities

$\geq, >$ great"or" "or" inequality

$\leq, <$ less th"and" "and" inequality

To solve absolute value inequalities:

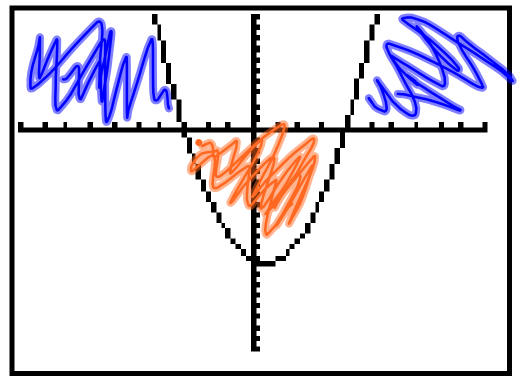
1. Isolate the absolute value on the left.
2. Decide if you have a "or" or "and" inequality
3. Write the inequalities.
4. Solve the inequalities.
5. Graph to find "and" intersection / "or union"
6. Check your answer in the original problem
7. Write the answer in interval notation.

$$|x+2| \leq 8$$

Solving Quadratic Inequalities

1. Make right side of the inequality 0
2. Decide if you have an "or" or "and" inequality
3. Solve the quadratic equation
4. Graph the quadratic and decide which values are above or below x-axis
5. Write in interval notation

$$x^2 - x - 12 > 0$$



Great"or" = Above x-axis

Less th"and" = Below x-axis

1.2

y-intercepts: where the graph crosses the y-axis and x = 0

x-intercepts: where the graph crosses the x-axis and y = 0

Function: when each domain value is paired with on one range value (no repeating x's)

- graphically: passes the vertical line test

Domain & Range (card)

Domain: x-values - input

read x's from left to rt. (smallest to largest)

*some functions have domain restrictions - can't divide by zero
to find: set the den. = 0 and solve for x. These are the restrictions.

can't have a neg. # in a sq. root
to find: set the radicand ≥ 0 and solve for x.

Range: y-values - output

read y's from bottom to top (smallest to largest)

Domain Restrictions:

1. Exclude any value that makes the denominator = 0

$f(x) = 2x^{4/3} - 2x^4$

$D: (-\infty, \infty)$

2. Exclude values that lead to the √ of a negative number

$f(x) = \sqrt{3-x}$

$D: (-\infty, 3]$

3. Taking the Log of a negative number

$f(x) = \frac{1}{\sqrt{3-x}}$

$D: (-\infty, 3)$

Asymptotes:

$x + 3$

vertical (VA): caused by dividing by 0

the graph approaches $-\infty$ or ∞

on each side of the asymptote

to find the asymptote set den = 0 and solve

end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the num and den. if

top heavy (OA):

bottom heavy (HA): y = 0

equal (HA): divide coefficients

oblique: (more later)

Increasing, Decreasing and Constant

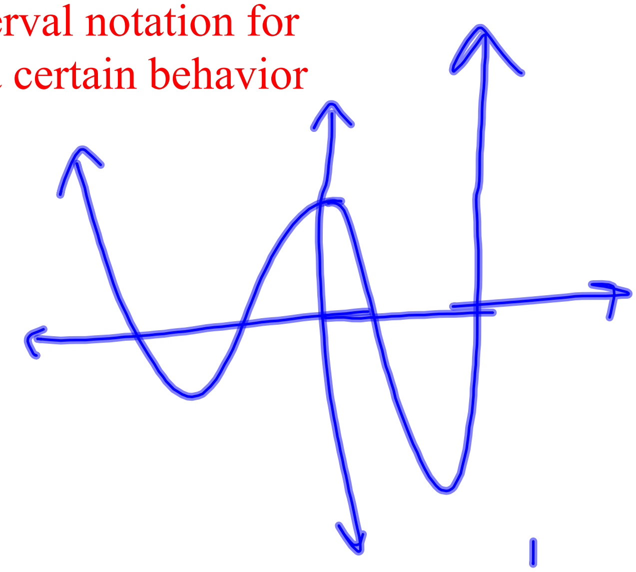
- as you move from left to right the y-values increase
- as you move from left to right the y-values decrease
- as you move from left to right the y-values do not change

this behavior is reported using interval notation for the x-values where the graph has a certain behavior

Extrema

maximums
- relative (local)
- absolute (upper bound)

minimums
- relative (local)
- absolute (lower bound)



Odd/Even/Neither Symmetry (card title)

$-(x^3)$ $y=-x^3$

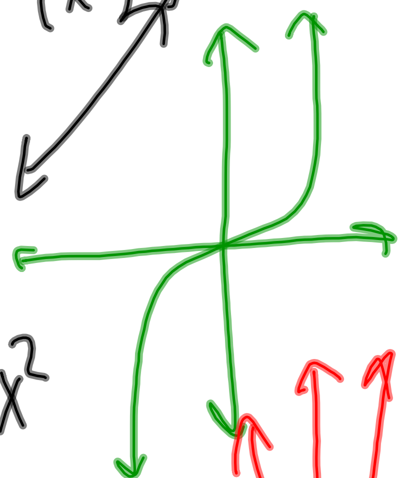
$y=x^3$

$y=(-x)^3$

$y=-x^3$

Odd $\boxed{f(-x) = -f(x)}$

symmetry with respect to the origin

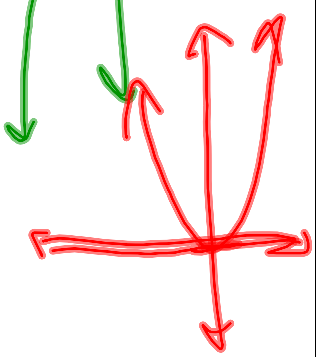


$y=x^2$

$y=(-x)^2 \Rightarrow y=x^2$

Even $\boxed{f(-x) = f(x)}$

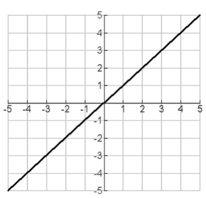
symmetry with respect to the y-axis



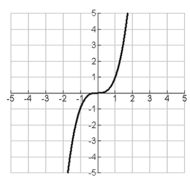
Neither

1.3

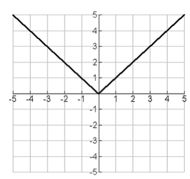
f(x) = x



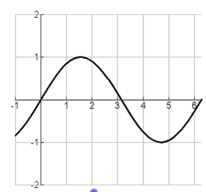
$f(x) = x^3$



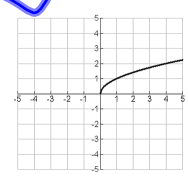
$f(x) = |x|$



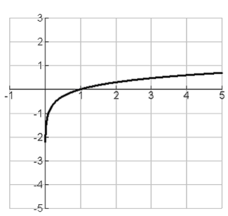
$f(x) = \sin x$



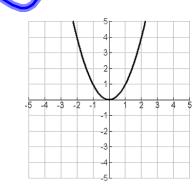
$f(x) = \sqrt{x}$



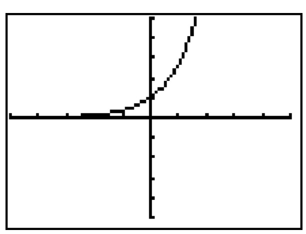
$f(x) = \ln x$



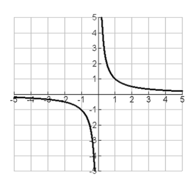
$f(x) = x^2$



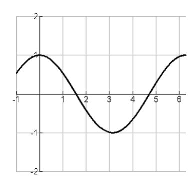
$f(x) = e^x$



$f(x) = \frac{1}{x}$



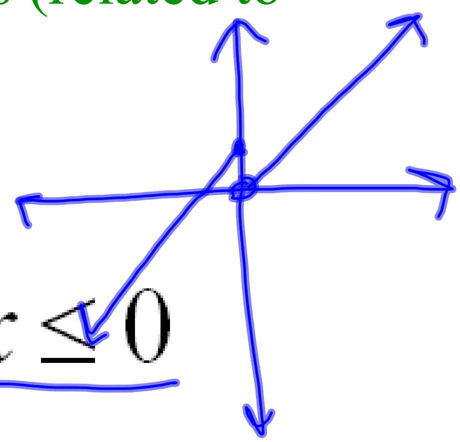
$f(x) = \cos x$



Piecewise Functions

certain pieces of the function have specific behavior

frequently: intervals (parts) of the domain are associated with different functions (related to continuity)



$$f(x)=\begin{cases} x+1 & if\ \ x\le 0 \\ x & if\ \ x>0 \end{cases}$$

1.4

Composition of Functions - defined

$$(f \circ g)(x) = f(g(x))$$

Finding the domain of a composition

$f(x) = x^2 - 1$

$g(x) = \sqrt{x}$

$(g \circ f)(x)$

$(f \circ g)(x)$

1. What is the domain of the first function?
2. Find the domain restrictions of the new function
3. Put them together

1.5

Finding an Inverse Algebraically (card)

Steps:

1. replace f(x) or relation name w/ y if not in that form

2. switch the x & y in the eq. (just x & y not signs, coefficients, or exponents)

3. Solve for y.

4. replace y with relation name (f^{-1} or g^{-1})

$$y = \frac{3x}{2x+7} \qquad x = \frac{3y}{\cancel{2y+7}}\cancel{(2y+7)} \quad (2y+7)$$

$$(2y+7)x = 3y$$

$$2xy + 7x = 3y$$
$$-3y \qquad -3y$$

$$2xy - 3y + 7x = 0$$
$$-7x \qquad -7x$$

$$2xy - 3y = -7x$$

$$\frac{y(2x-3)}{2x-3} = \frac{-7x}{2x-3}$$

$$y = \frac{-7x}{2x-3}$$

Domain changes
Range changes

$$y = \pm \Theta f(\pm \#(x \pm \Delta)) \pm ■$$

± if (–) reflection over x–axis (range Δ)

Θ vertical expansion or compression (range Δ)

Θ>1 expansion
Θ<1 compression

x's lie

± if (–) reflection over y–axis (domain Δ)

\# horizontal expansion or compression (domain Δ)

0<#<1 expansion
\#>1 compression

Δ translation left or right (domain Δ)

(+) left (-) right

■ translation up or down (range Δ)

(+) up (-) down

2.1

Quadratic Equations:

Standard Form:

$$y = ax^2 + bx + c$$

Graphing or Vertex Form:

$$y = a(x-h)^2 + k$$

complete the square!

vertex: (h,k)

axis of symmetry x=h

End Behavior (polynomial) 21

2.3

End Behavior is determined by the degree of the polynomial and the coefficient of the leading term. The mathematical notation is written using limits.

$$\lim_{x \to -\infty} f(x) =$$

left end

$$\lim_{x \to \infty} f(x) =$$

right end

Odd Degree: the left & right ends go in opp. directions

(+) coeff.

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

(-) coeff.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$

Even Degree: both ends go in the same direction

(+) coeff.

both up

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$

(-) coeff.

both down

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

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multiplicity

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the multiplicity.)

Straight intersection:
$(x - a)^1$ The power of the zero is 1.

Tangent intersection :
$(x - a)^{\text{even}}$ The power of the zero is even.

Inflection intersection: **(like a slide through)**
$(x - a)^{\text{odd}}$ The power of the zero is odd.