Domain & Range (card)

8

Domain: x-values - input

read x's from left to rt. (smallest to largest)

*some functions have domain restrictions - can't divide by zero

to find: set the den. = 0 and solve for x. These are the restrictions.

can't have a neg. # in a sq. root

to find: set the radicand ≥ 0 and solve for x.

Range: y-values - output

read y's from bottom to top (smallest to largest)

Asymptotes:

vertical (VA): caused by dividing by 0

the graph approaches $-\infty \;\; or \;\; \infty$

on each side of the asymptote

to find the asymptote set den = 0 and solve

end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the num and den. if

top heavy (OA):

bottom heavy (HA): y = 0

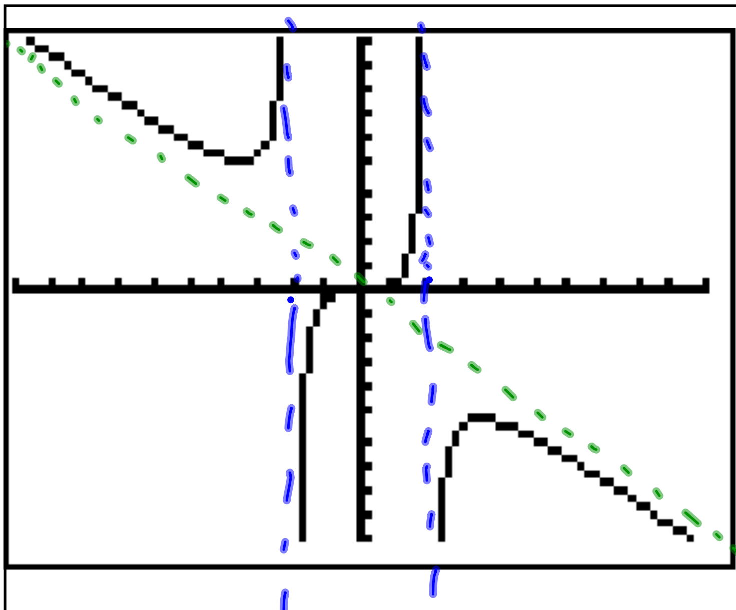
equal (HA): divide coefficients

oblique: (more later)

Increasing, Decreasing and Constant

- as you move from left to right the y-values increase
- as you move from left to right the y-values decrease
- as you move from left to right the y-values do not change

this behavior is reported using interval notation for the x-values where the graph has a certain behavior



$$f(x) = \frac{x^3}{4 - x^2}$$

2, -2

$$\begin{array}{r} -x \\ -x^2+4 \overline{)\, x^3 } \\ \underline{x^3 - 4x} \\ 0 - 4x \end{array}$$

Odd/Even/Neither Symmetry (card title)

$-x^3-2x$

$-(-x)^3-2(-x)$

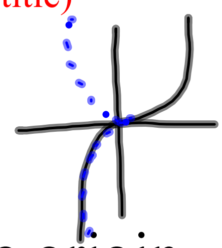
x^3+2x

$-(-x^3-2x)$

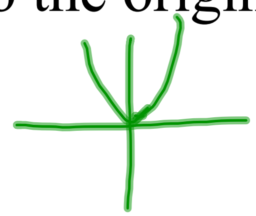
x^3+2x

11

Odd   f(-x) = - f(x) x^3



symmetry with respect to the origin



Even f(-x) = f(x) x^2

symmetry with respect to the y-axis

Neither

Inverses Functions & Relations (card)

Def: 2 relations f & g are inverses iff both of their compositions are the identity function (y=x)

this means f(g(x))=x & g(f(x))=x

- it does not mean they equal the same thing - they must equal x!!

Notation: $f^{-1}(x)$: inverse of f(x)

(not an exponent, f is the name of a function not a variable!!)

Finding an Inverse Algebraically (card)

Steps:

1. replace f(x) or relation name w/ y if not in that form
2. switch the x & y in the eq. (just x & y not signs, coefficients, or exponents)
3. Solve for y.
4. replace y with relation name^{-1} (f^{-1} or g^{-1})

Inverses - graphically (card)

Inverse relations are reflections of each other over the line y = x (identity function)

(2, 3) (-1, -2) (4, 7)

mirror images over y = x

(3, 2) (-2, -1) (7, 4)

so if g(x) and f(x) are inverses then every point (a,b) if f(x) will be reflected onto its mirror image (b,a) in g(x) and vise versa

Property of inverse relations: Suppose f & f^{-1} are inverse relations, then $f(a)=b$ iff $f^{-1}(b)=a$

Is the inverse a function????
How can I tell?

One-to-One (card)

- a function whose inverse is also a function

a function such that each x is paired with only one y and each y is paired with only one x

must pass the horizontal line test to be one to one

Transformations:

- helps us to understand the connections between the algebraic equation and its graph

- rigid - same shape and size (translation, reflection, rotation)
- non-rigid - distorts the shape by stretching and shrinking(dilation)

Domain changes
Range changes

Transformation Equation

$$y=\pm \Theta f(\pm \#(x \pm \Delta)) \pm \blacksquare$$

- order matters with combinations of transformations

- when using a table - multi. & divide first, add & subtract second

Domain changes
Range changes

$$y=\pm\Theta f(\pm\#(x\pm\Delta))\pm■$$

18

± if (–) reflection over x–axis (range Δ)

Θ vertical expansion or compression (range Δ)

$\Theta>1$ expansion
$\Theta<1$ compression

x's lie

± if (–) reflection over y–axis (domain Δ)

horizontal expansion or compression (domain Δ)

$0<\#<1$ expansion
$\#>1$ compression

Δ translation left or right (domain Δ)

(+) left (-) right

■ translation up or down (range Δ)

(+) up (-) down

Domain changes
Range changes

$$y=\pm\Theta f(\pm\#(x\pm\Delta))\pm■$$

18

$\pm$ if (–) reflection over x–axis (range Δ)

Θ vertical expansion or compression (range Δ)

- $\Theta>1$ expansion
- $\Theta<1$ compression

x's lie

$\pm$ if (–) reflection over y–axis (domain Δ)

horizontal expansion or compression (domain Δ)

- $0<\#<1$ expansion
- $\#>1$ compression

Δ translation left or right (domain Δ)

(+) left (-) right

■ translation up or down (range Δ)

(+) up (-) down

Polynomial Functions- Back

Name	Form	Degree
Zero Function	$f(x) = 0$	undefined
Constant Function	$f(x) = ax^0$ $a \neq 0$	0
Linear Function	$f(x) = ax + b$ $a \neq 0$	1
Quadratic Function	$f(x) = ax^2 + [bx + c]$ $a \neq 0$	2

2.3 Polynomial Functions

Polynomial Functions

Standard Form of a polynomial function:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots.. + \ldots.. a_0$$

term: each part of the polynomial - separated by + or (-)
leading term: term with the highest power or 1st term if written in standard form (poly. must be multiplied out to find this)
coefficient: number in front of the variable
constant: number w/o a variable

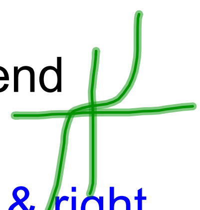
Degree: highest power found in any given term

End Behavior (polynomial) 21

End Behavior is determined by the degree of the polynomial and the coefficient of the leading term. The mathematical notation is written using limits.

$$\lim_{x \to -\infty} f(x) =$$

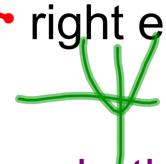
left end



$$\lim_{x \to \infty} f(x) =$$

right end

x^2



Odd Degree: the left & right ends go in opp. directions

(+) coeff.

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

(-) coeff.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$

Even Degree: both ends go in the same direction

(+) coeff.
both up

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$

(-) coeff.
both down

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

Zeros(roots) and Mulitiplicity

22

zero $\boxed{7}$ factor $\underline{(x-7)}$

Zeros: solutions for x when y = 0

can be found in the **factors** $(x - a)$ of the polynomial.

How do we find the zeros??

- factor
- quadratic formula
- use the calculator

$f(x) = x^2 + 6x + 8$

$f(x) = (x+4)(x+2)$

$-4, -2$

$0 = x^2 + 6x + 8$

What are the differences between factors and zeros???

multiplicity

22

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the multiplicity.)

Straight intersection:
$(x - a)^1$ The power of the zero is 1.

Tangent intersection :
$(x - a)^{even}$ The power of the zero is even.

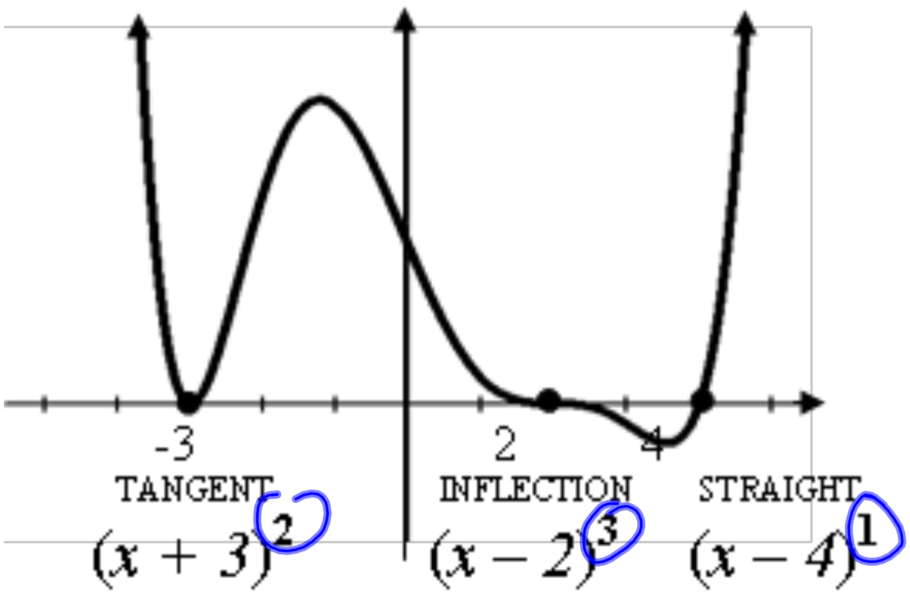
Inflection intersection: **(like a slide through)**
$(x - a)^{odd}$ The power of the zero is odd.

$$y = (x + 3)^2 (x - 2)^3 (x - 4)$$

What are the zeros??

What is the mulitplicity (power) of the zero??

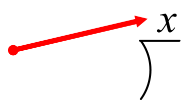
How will it intersect the x-axis??



Dividing Polynomials - Long Division #23

Steps: 1. Write as a division problem w/ dividends & divisor in descending order, leaving spaces for missing terms in the dividend (0x)

2. Divide leading terms and write the result above the 1st term in the dividend



3. Multiply the result from #2 by the divisor & write the product under the dividend

4. Put ( ) around result from #3, distribute the subtraction sign & then add

5. Bring down remaining terms & repeat until there are no remaining terms in the dividend

6. Answer can be written in several ways - see back

$$(x^4 - 2x^3 + 3x^2 - 4x + 6) \div (2x + x^2 + 1)$$

Dividing Polynomials - Synthetic division: #24

Can only be used to divide by a linear function

steps:

1. Write the terms of the dividend in descending order. Write the coeff. of the dividend in the first row using zeros for any missing terms not found in the dividend.

2. Write the zero, r, of the divisor (x-r),in the box.

3. Drop the 1st coeff. to the last row.

4. Multiply 1st coeff. by r & put product under the 2nd coeff.

5. Add product from #4 to 2nd coeff. & write the sum in the last row.

6. Repeat #4 & #5 until all coeff. have been used.

7. Write answer by putting variables behind the #'s in the last row. Start with 1 degree less than the dividend polynomial.

$$\frac{x^3 - 5x^2 + 3x - 2}{x + 1}$$

-1	1	-5	3	-2
	↓	-1	6	-9
	1	-6	9	-11 Remainder

$$x^2 - 6x + 9 - \frac{11}{x+1}$$

What is the significance of the answers??

#25

Remainder Theorem & Factor Theorem

Remainder Theorem: $f(k)$ = remainder

this means - evaluate the function for the value of the suspected zero (plug it in for x)

Factor Theorem: if the remainder is 0 then you have found a root!!! ($f(k) = 0$)

Rational Root Theorem: #138

if all coefficients are integers and the constant is not 0, then all possible rational roots are:

$2x^4 + 10x^3 - 9x^2 - 15$

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}} \qquad \pm \frac{1, 3, 5, 15}{2, 1}$$

$\pm \frac{1}{2}, 1, \frac{3}{2}, 3, \frac{5}{2}, 5, \frac{15}{2}, 15$

or $x = \pm \frac{p}{q}$ when p = factors of constant
q = factors of leading coefficient

2.5 Complex zeros & Fundamental Thm of Algebra

Top Half of Card 27

#27

Fundamental Thm of Alg: an nth degree polynomial will have n zeros

(may be a combination of real and complex & some zeros may be repeated)

Odd functions will always have at least one real zero -why??

Complex Conjugates: complex factors come in conjugate pairs
(if 3i is a zero, - 3i is also)

Use the given zero to find the remaining zeros and write a linear factorization:

$$3 - 2i;\ \ x^4 - 6x^3 + 11x^2 + 12x - 26$$

Asymptotes:

check for holes before VA!! (by reducing the fraction if possible)

vertical (VA): caused by dividing by 0

the graph approaches $-\infty \quad or \quad \infty$
on each side of the asymptote

to find the asymptote set den = 0 and solve

end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the num and den. if top heavy (OA):
bottom heavy (HA): y = 0
equal (HA): divide coefficients

oblique: (more later)

Oblique/Slant Asymptotes

#29

top heavy rational functions have oblique asymptotes (end behavior models)

to find the degree of the end behavior model - divide the leading terms and reduce

the ends of $\dfrac{3x^5 - 4x^2 + 5}{2x^3 - 5x + 4}$ will behave like $\dfrac{3x^5}{2x^3} = \boxed{\dfrac{3x^2}{2}}$

to find the actual asymptote: divide the fractions

2.7 Solving Rational Equations

31

Rational Equation: an eq. (has an =) made up of 1 or more rational expressions

steps -

- find restrictions (why do I have restrictions?)
- Find the LCD
- Multiply each term in eq. by LCD to clear fractions
- solve the equation
- check for extraneous solutions

3.1 Exponential Functions

#33

$$y = a \bullet b^x \qquad a \neq 0,\ b \neq 1$$

$$b > 0$$

Used for growth and decay of: bacteria, carbon, populations

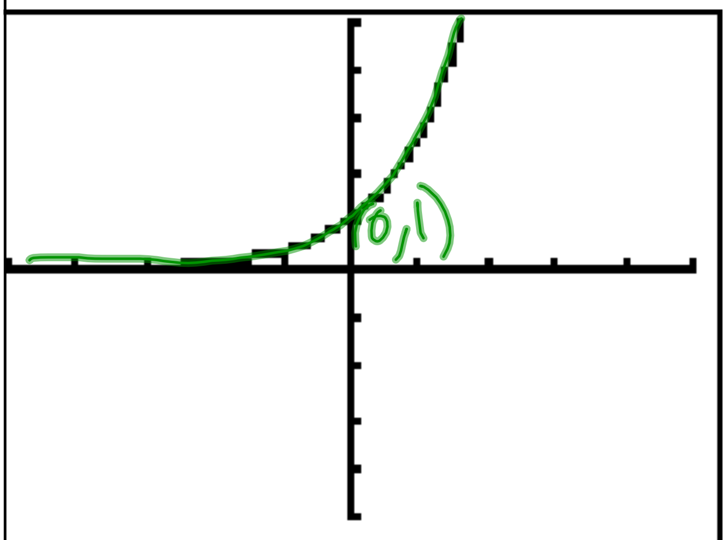
$$y = a_0 \bullet b^x$$

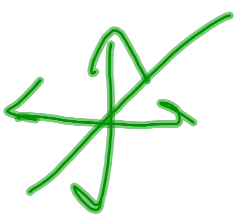
a_0 is the inital value

b is the base

$b > 1$ growth

$0 < b < 1$ decay





Domain (-∞,∞)
Range (0,∞)
Continuous yes
Increasing (-∞,∞)
Decreasing ___
Constant ___
Left End to 0
Right End ∞
Symmetry neither
x-intercepts none
y-intercepts (1,0)
VA ___
HA y=0
Bounded Below
Extrema ___

$$f(x) = a_0 \bullet b^x$$

$$f(x) = a_0 \bullet (1 \pm r)^x$$

When looking at percent increase or decrease - the base is expressed as 100% + or - the % change.

$$f(x) = 2 \bullet .73^x$$

Is this an increase or decrease?

By what %?

3.3 Log Functions and their Graphs

\# 35

Logarithms

Def of Logarithm: if $b > 0,\ b \neq 0,\ n > 0,$ there is a number p such that $\log_b n = p \ \ iff \ \ b^p = n$

exponent, power

$n = b^p \qquad p = \log_b n$

base - must match

a number or variable

$y = 2^x \qquad x = \log_2 y$

p is the only value that can be negative

$3^2 = 9$

$2 = \log_3 9$

$16^{\frac{1}{2}} = 4$

$\log_{16} 4 = \frac{1}{2}$

$2^{-3} = \frac{1}{8}$

$-3 = \log_2 \frac{1}{8}$

Basic Properties of Logarithms

For 0<b ≠ 1, x > 0, and any real number y,

$\log_b 1 = 0$ because

$\log_b b = 1$ because

$\log_b b^y = y$ because

$b^{\log_b x} = x$ because

36

Properties of Logs

for all positive #'s M, N, b, and x:

Product Rule

$$\log_b M \bullet N = \log_b M + \log_b N$$

Quotient Rule

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

Power Rule

$$\log_b M^x = x \log_b M$$

Change of Base

#37

$$\log_b a = \frac{\log_x a}{\log_x b} \quad or \quad \frac{\ln a}{\ln b}$$

3.5 Solving Exp & Log Equations

w/ matching bases

#38

If the bases are the same -

Properties of Logarithms & exponents

Switch Forms

Graphing

★ one-to-one property

If $b^u = b^v$, then $u = v$

If $\log_b u = \log_b v$, then $u = v$

Solving Exp & Log Equations

w/o matching bases

#37-back

When the bases don't match:

you must ~~use~~ un convert to the other form:

use Inverse properties

$\log_b a = x \quad b^x = a$

$\log a = x \quad 10^x = a$

$\ln a = x \quad e^x = a$

Interest

#39

Compounded Annually: $$A = P(1+r)^n$$

Compounded k Times per year: $$A = P(1+\frac{r}{k})^{kt}$$

Compounded Continuously: $$A = Pe^{rt}$$