Complex Number System

standard form of a complex number

$$a + bi$$

real #'s

$b = 0$

Imaginary #'s

pure imaginary

$a = 0$

non-pure

$a \neq 0$

If

$b=0$ the # is real

$b\neq0$ the # is imaginary

$a = 0$ the # is pure imaginary

Def of Equal Complex #'s: $a+bi = c+di$ iff $a = c$ and $b = d$
so 2 complex #'s are equal if the real parts are equal & imaginary parts are equal

What are the real and imaginary components

-2 + 4i → IMAGINARY

REAL

0 + -7i → IMAGINARY

REAL

Find x and y:

$3 + 5i = x + yi$

$x = 3$

$y = 5$

$x - 5i = (2 - i) + (4 + 2yi)$

$x - 5i = 6 - i + 2yi$

$x - 5i = 6 + i(-1 + 2y)$

$x = 6$

$-5 = -1 + 2y$

$+1 \quad +1$

$-4 = 2y$

$\frac{-4}{2} = \frac{2y}{2}$

$y = -2$

Complex Numbers

~~#22~~ #21

$\sqrt{-1} = i$ $\qquad$ $\left(\sqrt{-1}\right)^2 = -1$

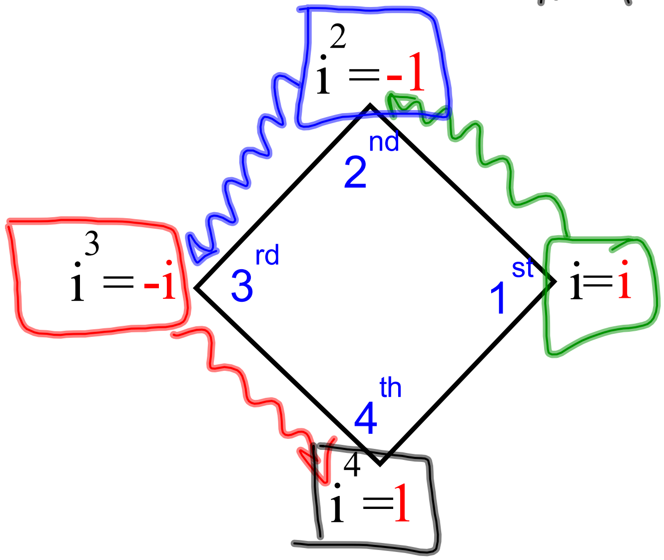
Powers of i

$i = i^5 = i^9$

$i^2 = i^6 = i^{10}$

$i^3 = i^7 = i^{11}$

$i^4 = i^8 = i^{12}$



Pattern repeats every 4 iterations

$$i^{23} = i^{20} \cdot i^3 = i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^3 = 1 \cdot -i = -i$$

$$i^{40} = 1$$

$$i^{38} = i^{36} \cdot i^2 = i^2 = \boxed{-1}$$

$$i^3 + i^5 = -i + i = 0$$

Simplify:

1. $\sqrt{-81}$ $= \pm 9i$

2. $\sqrt{-121x^5}$ $= \pm 11 i x \cdot x \sqrt{x} = \pm 11 i x^2 \sqrt{x}$
$x^2 \cdot x^2 \cdot x$

3. $8i \bullet 3i = 24i^2 = -24$

4. $\sqrt{-5} \bullet \sqrt{-20} = \sqrt{100} = \pm 10$

5. $i^{12} = 1$

6. i^{43} $i^{40} \cdot i^3 = 1 \cdot -i = -i$

7. Solve $x^2 + 81 = 0$
$-81 \quad -81$
$\sqrt{x^2} = \sqrt{-81}$
$x = \pm 9i$

Operations w/ Complex #'s

add/subtract: combine like terms (real w/real and imaginary w/imaginary)

Multiply: FOIL

Divide: factor or use conjugates to simplify (remember *i* is a radical and can't stay in the denominator)

Simplify:

$(-4 - i) - (5+2i)=$

$-4 - i - 5 - 2i$

$-9 - 3i$

$(3 - 2i)(4 + 3i)$

$12 + 9i - 8i - 6i^2$

$= 18 + i$

$(6 - 2i^2) - (5+2i)=$

$6 - 2i^2 - 5 - 2i$

$6 + 2 - 5 - 2i \quad 3 - 2i$

$(-2+3i)(\sqrt{-9}+3)$

$-2\sqrt{-9} - 6 + 3i\sqrt{-9} + 9i$

$-6i - 6 + 9i^2 + 9i$

$-15 + 3i$

$\left(\sqrt{2}+3i\right)-\left(\sqrt{3}+3i\right)=$

$\sqrt{2} + 3i - \sqrt{3} - 3i$

$\sqrt{2} - \sqrt{3}$

Conjugates

Radical Conjugates: 2 radical expressions $a+c\sqrt{d}$ and $a-c\sqrt{d}$ are conjugates because when multiplied together the radicals are eliminated

Complex Conjugates: 2 complex #'s a + bi and a - bi are conjugates because when multiplied all imaginary parts are eliminated

$\left(2-\sqrt{3}\right)$ $\left(-5+\sqrt{5}\right)$

3+2i
(3 - 2i)

-4 - 3i
(- 4 + 3i)

Simplify:

$$\frac{3i}{2-4i}\cdot\frac{(2+4i)}{(2+4i)}$$

$$\frac{6i+12i^2}{4+\cancel{8i}-\cancel{8i}-16i^2}$$

$$\frac{6i-12}{4+16}$$

$$\frac{6i-12}{20}$$

$$\frac{3i-6}{10}$$

$$\frac{3+7i}{2i}\cdot\frac{(-2i)}{(-2i)}$$

$$\frac{-6i-14i^2}{-4i^2}$$

$$\frac{-6i+14}{4}$$

$$\frac{-3i+7}{2}$$

$$i^{-27}$$

$$\frac{1}{i^{27}}$$

$$\frac{1}{\cancel{i^{24}}\cdot i^3}$$

$$\frac{1}{-i}\frac{(i)}{(i)}$$

$$\frac{i}{-i^2}$$

$$\frac{i}{-1}$$

$$1$$

$$i$$

Complex solutions of quadratic equations

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$ = Discriminant

$b^2 - 4ac > 0$ = 2 Real solutions

$b^2 - 4ac = 0$ = 1 Real solution

$b^2 - 4ac < 0$ = 2 complex solutions

Solve for x: $x^2 + 3x - 4 = x - 9$

$-x+9 \quad -x+9$

$a=1$

$b=2$

$c=5$

$x^2+2x+5=0$

$x = \dfrac{-2 \pm \sqrt{2^2-4(1)(5)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4-20}}{2}$

$x = \dfrac{-2 \pm \sqrt{-16}}{2}$

$x = \dfrac{-2 \pm 4i}{2}$

$\boxed{x = -1 \pm 2i}$

$\sqrt{-17}$

$\sqrt{17}i$

$i\sqrt{17}$