What is a graph?
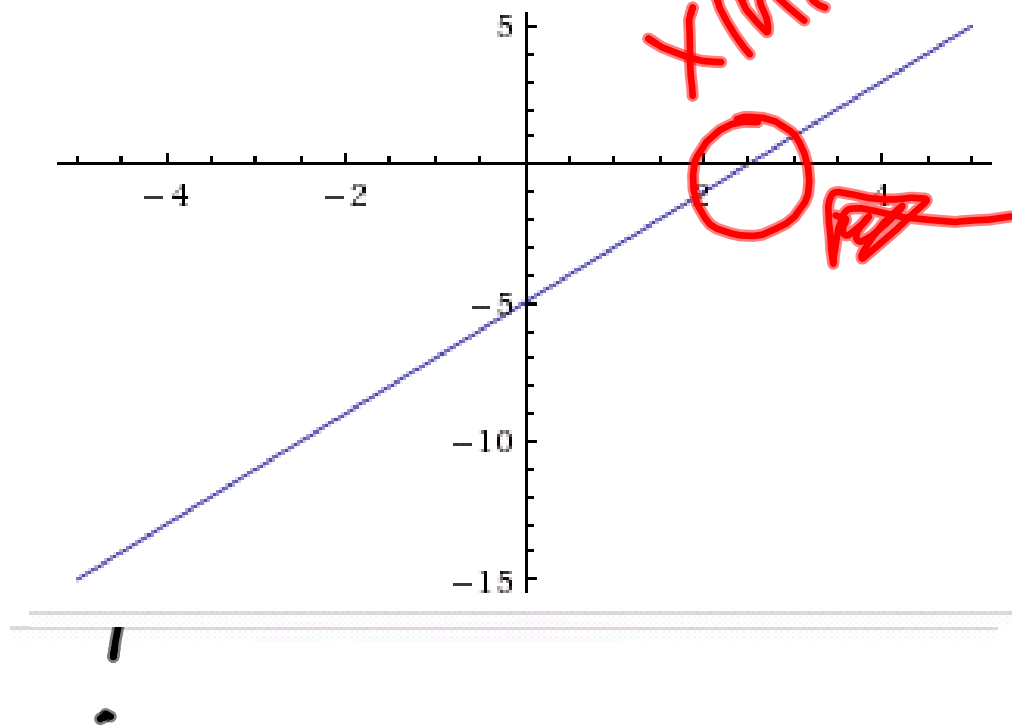
$$y = 2x - 5$$

XIntercept
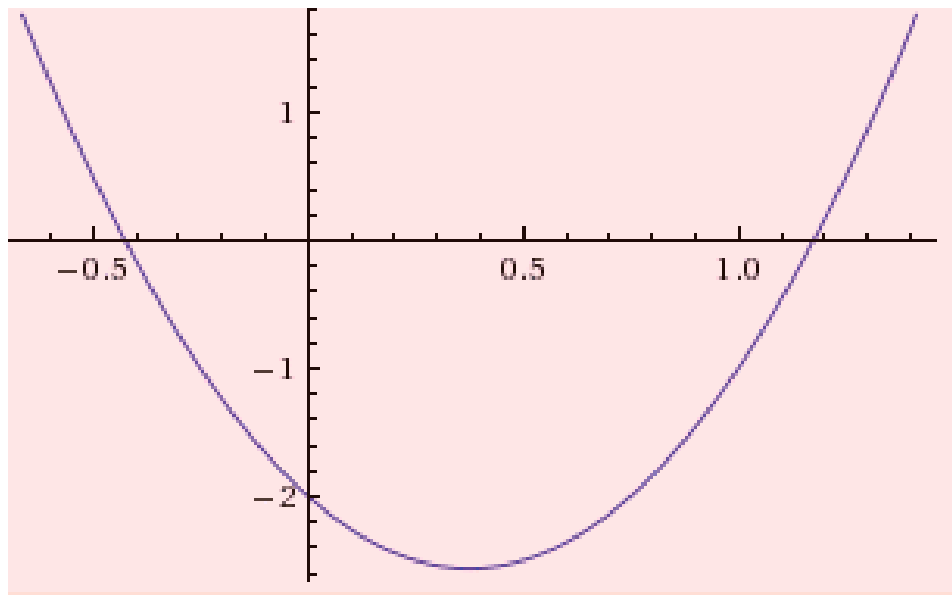

If we want to solve $0 = 2x - 5$ on a graph what point are we looking for?

≈ 2.5

$$y = 2x - 5$$
$$0 = 2x - 5$$
$$+5 \qquad +5$$
$$\frac{5}{2} = \frac{2x}{2}$$
$$2.5 = \frac{5}{2} = x$$

Solve $2x^2 - 3x - 2 = 0$ Graphically

≈ 2
$\approx -\frac{1}{2}$

$2x(y) + y$
$y(2x+1)$

Verify Algebraically

$2x^2 - 3x - 2 = 0$ $-4, 1$
$a = 2$ $ac = -4$
$b = -3$ $xy = -4$
$c = -2$ $x + y = -3$

$(2x^2 - 4x) + (x - 2) = 0$

$2x(x - 2) - (x - 2) = 0$

$(x - 2)(2x + 1) = 0$

$x - 2 = 0$ $2x + 1 = 0$
$x = 2$ $-1 = -1$
                     $2x = -1$
                     $\frac{2x}{2} = \frac{-1}{2}$
                     $x = -\frac{1}{2}$

Quadratic Equations:

A Quadratic equation in x is one that can be written in the form

$$ax^2 + bx + c = 0 \quad a,b,c \ni \mathbb{R}, a \neq 0$$

Ways to solve Quadratic equations

1. Factoring
2. extracting square roots $(ax+b)^2 = C$
3. Completing the square
4. Quadratic Formula

Extracting Square Roots $(ax+b)^2 = c$

Example: $(2x-1)^2 = 9$

$$2x - 1 = \pm 3$$

$$2x - 1 = 3$$
$$+1 \quad +1$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

$$2x - 1 = -3$$
$$+1 \quad +1$$
$$\frac{2y}{2} = \frac{-2}{2}$$
$$x = -1$$

$$\boxed{x = 2, -1}$$

Completing the Square

$$x^2 + bx = c \quad \text{add} \quad (\frac{b}{2})^2 \quad \text{to both sides}$$

$$x^2 + bx + (\frac{b}{2})^2 = c + (\frac{b}{2})^2$$

Find the perfect Square

$$(x + \frac{b}{2})^2 = c + \frac{b^2}{4}$$

Solve by extracting square roots

Example

$$4x^2 - 20x + 17 = 0$$

$$y = 2x^2 + 6x - 5 = 0$$

$$(x-a)(x-a)$$

$$x^2 - ax - ax + a^2$$
$$x^2 - \frac{2ax}{2} + a^2$$

$$y + \frac{9}{2} = 2\left(x^2 + 3x + \frac{9}{4}\right) - 5$$

$$y + \frac{9}{2} = 2\left(x + \frac{3}{2}\right)^2 - 5$$
$$+5 \qquad +5$$

$$\frac{y + \frac{19}{2}}{2} = \frac{2\left(x + \frac{3}{2}\right)^2}{2} = \frac{1}{2}y + \frac{19}{4} = \left(x + \frac{3}{2}\right)^2$$

Solve by the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ax^2 + bx + c = 0$$

Example:

$$\frac{3x^2 - 6x = 5}{-5}$$

$$3x^2 - 6x - 5 = 0$$

$$a = 3$$
$$b = -6$$
$$c = -5$$
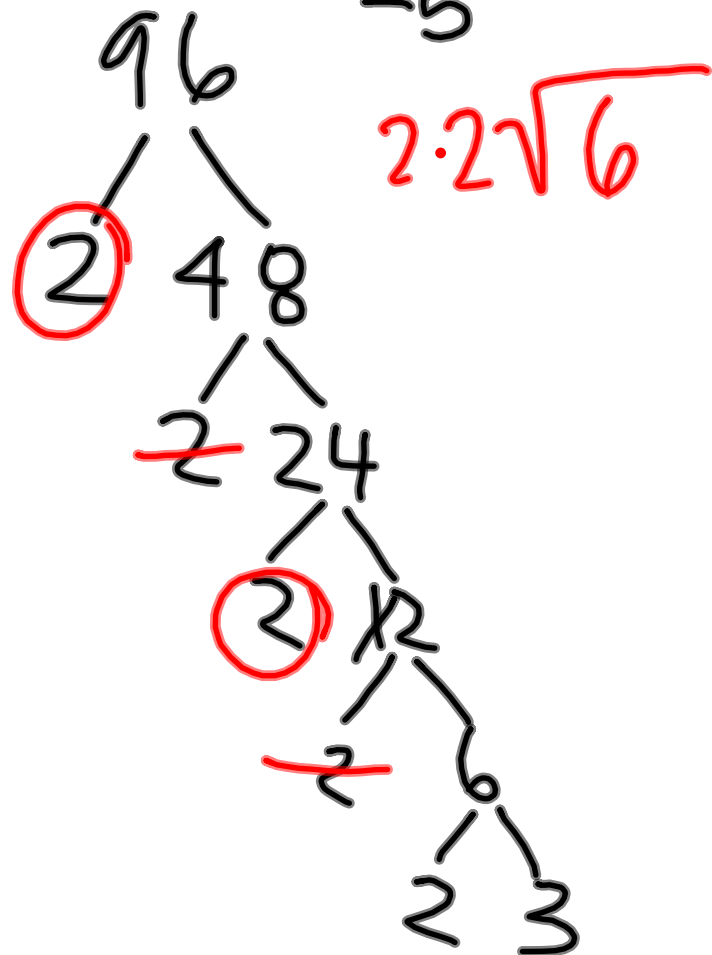
$$x = \frac{6 \pm \sqrt{36 - 4(3)(-5)}}{2(3)}$$

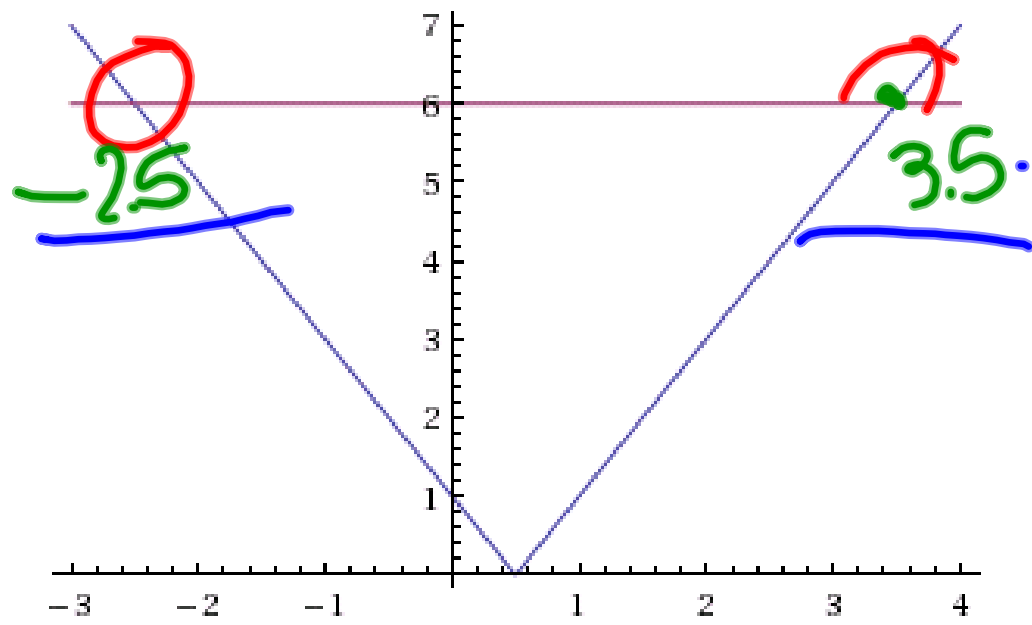$$x = \frac{6 \pm \sqrt{36 + 60}}{6}$$

$$x = \frac{6 \pm \sqrt{96}}{6}$$

$$96$$

$$2 \cdot 2\sqrt{6}$$

$$② \quad 48$$

$$2 \quad 24$$

$$② \quad 12$$

$$2 \quad 6$$

$$2 \quad 3$$

$$= \frac{6 \pm 4\sqrt{6}}{6} = \boxed{\frac{3 \pm 2\sqrt{6}}{3}}$$

Solve Using Intersections

Solve $|2x-1|=6$ by graphing

$y=|2x-1|$ $y=6$

Verify Algebraically

$|2x-1|=6$

$2x-1=\pm 6$

-2.5

3.5.

$2x-1=6$
$\quad +1 \; +1$
$2x=7$
$\frac{2x}{2}=\frac{7}{2}$
$x=\frac{7}{2}=3.5$

$2x-1=-6$
$\quad +1 \quad +1$
$2x=-5$
$\frac{2x}{2}=\frac{-5}{2}$
$x=\frac{-5}{2}=-2.5$