9.5 Series

#71

Summation

$$a_1 + a_2 + a_3 + \ldots + a_n$$

(how do we write the sum of long lists of numbers?)

$\sum$ sigma means summation

where to end

equation

Summation notation: $\sum_{k=1}^{n} \boxed{a_k} = \underline{a_1 + a_2 + a_3 + \ldots + a_n}$

where to start

$$\sum_{k=1}^{5} 3k = 3(1)+3(2)+3(3)+3(4)+3(5) = 45$$

$$\sum_{k=5}^{8} k^2 = 5^2+6^2+7^2+8^2 = 174$$

$$\boxed{a_n = a_1 + (n-1)d}$$

$$29 = 2 + (n-1)3$$

$$a_n = 3n-1$$

$$-2 \quad -2$$

$$2+3n-3$$

$$\frac{27}{3} = \frac{3(n-1)}{3}$$

$$a_n = 3n-1$$

$$9 = n-1$$

$$a_n =$$

$$+1 \quad +1$$

$$n = 10$$

2+5+8+11+...+29

$d=3 \quad a_1=2 \quad n=?$

$$\sum_{k=1}^{10} 3k-1$$

Sum of a Finite Arithmetic Sequence:

#72

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

$$= \frac{n(a_1 + a_n)}{2}$$

Memorize For Test *

where it ends

Common difference

$$= \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

First term

A theater has 8 seats in the first row. Each successive row has 2 additional seats. The top row has 24 seats. How many seats in a section?

$a_1 = 8 \quad d = 2 \quad a_n = 24$

$a_n = a_1 + (n-1)d$

$24 = 8 + (n-1)2$

$-8 \quad -8$

$\frac{16}{2} = \frac{(n-1)2}{2}$

$8 = n - 1$

$+1 \quad +1$

$9 = n$

$\frac{n}{2}(2a_1 + (n-1)d)$

$\frac{9}{2}(2(8) + (9-1)(2))$

$= \frac{9}{2}(16 + 16)$

$= \frac{9}{2}(32)$

$= \boxed{144}$

Sum of a Finite Geometric Sequence: #73

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

memorize * for test

$$= \frac{a_1\left(1 - r^n\right)}{1 - r}$$

Find the sum of: $a_n = a_1 \cdot r^{n-1}$

$$\frac{a(1-r^n)}{1-r}$$

$$4, \frac{-4}{3}, \frac{4}{9}, \frac{-4}{27}, \ldots\ldots, 4\left(\frac{-1}{3}\right)^{10}$$

$a = 4,\ r = -\frac{1}{3},\ n = 11$

$$\frac{4(1-(-1/3)^{11})}{1-(-1/3)} \approx 3$$

3+6+12+....+12,288

$r = 2 \quad a_1 = 3$

$a_n = a_1 \cdot r^{n-1}$

$$\frac{12{,}288}{3} = \frac{3(2)^{n-1}}{3}$$

$4096 = 2^{n-1}$

$2^{12} = 2^{n-1}$

$12 = n-1$

$n = 13$

$$\frac{3(1-(2)^{13})}{1-2}$$

$= 24{,}573$

Series:

#74

def: sum of the terms in a sequence

sum: usually a total of a finite number of items added together

*partial sums: the sums of a specific number of terms in the infinite sequence

(these are used to talk about the infinite series) as you look at the partial sums, they approach a specific number

$$\lim_{n\to\infty}\sum_{k=1}^{n} a_k = S$$

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\ldots$$

$S_1 = 1$

$S_2 = 1.5$

$S_3 = 1.75$

$S_4 = 1.825$

$S_5 = 1.9$

this is called a converging series

your partial sums could approach ∞ , $-\infty$,

or the limit doesn't exist because the numbers oscillate

$$\lim_{n\to\infty}\sum_{k=1}^{n} a_k = \infty, -\infty, \text{doesn't exist}$$

3 + 6 + 9 + 12 +...

$S_1=3$ $S_2=9$ $S_3=18$

2 - 2 + 2 - 2 +....

$S_1=2$ $S_2=0$ $S_3=2$ $S_4=0$

this is called a diverging series

Infinite Geometric Series: #75

$$\sum_{k=1}^{\infty} a \bullet r^{k-1} = S$$ converges if $|r| < 1$

it will converge to: $$S = \frac{a}{1-r}$$

a = first term
r = common ratio

Determine if the geometric series converges or diverges.
If it converges, find its sum.

$$\sum_{n=0}^{\infty}\left(-\frac{4}{5}\right)^n$$ $\left|-\frac{4}{5}\right| < 1$ Converges $\frac{1}{1+\frac{4}{5}} = \boxed{\frac{5}{9}}$

$$\sum_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^n$$ $\left|\frac{\pi}{2}\right| = 1.57$ Diverges

$$\sum_{n=0}^{\infty}2\left(\frac{1}{5}\right)^n$$ Converges $\frac{2}{\frac{5}{5}1-\frac{1}{5}}$ $\frac{2}{\frac{4}{5}} = \boxed{\frac{10}{4}}$