9.5 Series

Summation

\[ a_1 + a_2 + a_3 + \ldots + a_n \]

(how do we write the sum of long lists of numbers?)

\[ \sum \text{ sigma} \]

\[ \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n \]
\[ \sum_{k=1}^{5} 3k = 3(1) + 3(2) + 3(3) + 3(4) + 3(5) = 45 \]

\[ \sum_{k=5}^{8} k^2 = 5^2 + 6^2 + 7^2 + 8^2 = 174 \]

2 + 5 + 8 + 11 + ... + 29

\[ a = 3 \quad a_1 = 2 \quad n = ? \]

\[ \sum_{k=1}^{10} 3k - 1 \]

\[ a_n = a_1 + (n-1)d \]

\[ a_{29} = 2 + (n-1)3 \]

\[ a_{27} = \frac{3(n-1)}{2} \]

\[ a = n - 1 \]

\[ a_n = 3n - 1 \]

\[ n = 10 \]
Sum of a Finite Arithmetic Sequence:

\[ \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n \]

\[ = \frac{n(a_1 + a_n)}{2} \]

\[ = \frac{n}{2} \left( 2a_1 + (n-1)d \right) \]

- Memorize
- For Test
- First term
- Common difference
A theater has 8 seats in the first row. Each successive row has 2 additional seats. The top row has 24 seats. How many seats in a section?

\[
\begin{align*}
a_1 &= 8 \quad d = 2 \quad a_n = 24 \\
a_n &= a_1 + (n-1)d \\
24 &= 8 + (n-1)2 \\
-8 &= (n-1)2 \\
8 &= n - 1 \\
 n &= 9 \\
\frac{n}{2}(2a_1 + (n-1)d) &= \frac{9}{2}(2(8) + (9-1)(2)) \\
&= \frac{9}{2}(16 + 16) \\
&= \frac{9}{2}(32) \\
&= 144
\end{align*}
\]
Sum of a Finite Geometric Sequence:  

\[ \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n \]

\[ = \frac{a_1 \left(1 - r^n\right)}{1 - r} \]

*memorize for test*
Find the sum of: \[ a_n = a_1 \cdot r^{n-1} \]
\[
4, \frac{-4}{3}, \frac{4}{9}, \frac{-4}{27}, \ldots, 4 \left( \frac{-1}{3} \right)^{10}
\]
\[ a = 4, \quad r = -\frac{1}{3}, \quad n = 11 \]
\[
\frac{a(1-r^n)}{1-r} = \frac{4 \left(1 - \left(-\frac{1}{3}\right)^{11}\right)}{1 - \left(-\frac{1}{3}\right)} \approx 3
\]

\[
3+6+12+\ldots+12,288
\]
\[ r = 2, \quad a_1 = 3 \]
\[ a_n = a_1 \cdot r^{n-1} \]
\[
\frac{12,288}{3} = \frac{3(2)^{11}}{3} = 24,573
\]
\[
4096 = 2^{12} = 2^{n-1}
\]
\[ 12 = n-1 \]
\[ n = 13 \]
Series: 

**def**: sum of the terms in a sequence

**sum**: usually a total of a finite number of items added together

**partial sums**: the sums of a specific number of terms in the infinite sequence

( these are used to talk about the infinite series )
as you look at the partial sums, they approach a specific number

\[ \lim_{n \to \infty} \sum_{k=1}^{n} a_k = S \]

\[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \]

\[ S_1 = 1 \quad S_4 = 1.875 \]
\[ S_2 = 1.5 \quad S_5 = 1.9 \]
\[ S_3 = 1.75 \]

this is called a converging series
your partial sums could approach $\infty$, $-\infty$, or the limit doesn't exist because the numbers oscillate

$3 + 6 + 9 + 12 + \ldots$

$s_1 = 3 \ s_2 = 9 \ s_3 = 18$

$2 - 2 + 2 - 2 + \ldots$

$s_1 = 2 \ s_2 = 0 \ s_3 = 2 \ s_4 = 0$

this is called a diverging series
Infinite Geometric Series:

\[ \sum_{k=1}^{\infty} a \cdot r^{k-1} = S \]

it will converge to:

\[ S = \frac{a}{1-r} \]

converges if \(|r| < 1\)

\#75

a = first term

r = common ratio
Determine if the geometric series converges or diverges. If it converges, find its sum.

\[ \sum_{n=0}^{\infty} \left( -\frac{4}{5} \right)^n \]

\[ \left| -\frac{4}{5} \right| < 1 \quad \text{Converges} \quad \frac{1}{1+\frac{4}{5}} = \frac{5}{9} \]

\[ \sum_{n=1}^{\infty} \left( \frac{\pi}{2} \right)^n \]

\[ \left| \frac{\pi}{2} \right| = 1.57 \quad \text{Diverges} \]

\[ \sum_{n=0}^{\infty} 2 \left( \frac{1}{5} \right)^n \]

\[ \left| \frac{1}{5} \right| = \frac{1}{5} \quad \text{Converges} \quad \frac{2}{\frac{1}{5}} = \frac{10}{4} \]