Most of the formulas we have today are the result of mathematicians noticing patterns. The binomial theorem is one such pattern.

Expand $(a + b)^n$ for n = 0,1,2,3,4, &5

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$
\begin{array}{ccccc}
 & & {}_0C_0 & & \\
 & {}_1C_0 & & {}_1C_1 & \\
{}_2C_0 & & {}_2C_1 & & {}_2C_2
\end{array}
$$

Binomial Coefficient

The binomial coefficients that appear in the expansion of $(a+b)^n$ are the values of ${}_nC_r$ for r = 1, 2, 3, 4, ..., n.

A classical notation for ${}_nC_r$ is : $\binom{n}{r}$ meaning "n choose r"

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad \binom{15}{11}$$

Expand pascal's triangle to find the next row.

Find the coefficient of x^{10} in the expansion of $(x + 2)^{15}$

The Binomial theorem #67

For any positive integer n,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

where $\binom{n}{r} = {}_nC_r = \dfrac{n!}{r!(n-r)!}$

$$\left(x^2+y\right)^3=$$

$$\left(3x-y^2\right)^5$$