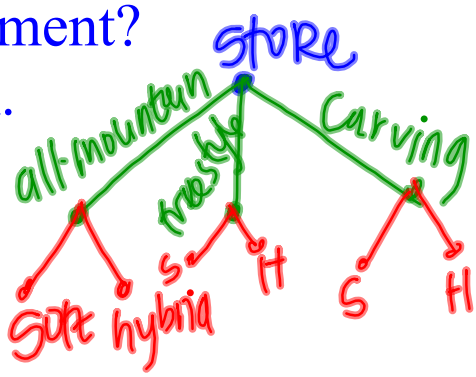
9.1 Counting Principle and Permutations

A sporting goods store offers 3 types of snowboards (all-mountain, freestyle, carving) and 2 types of boots (soft or hybrid). How many choices are there for snowboarding equipment?
Draw a tree diagram.



6 choices

Fundamental Counting Principle:
If event M can occur in m ways and is followed by an independent event N that occur in n ways, then the events M followed by the event N can occur in m * n ways.

So if we use the previous example - what would we have??

$3 \cdot 2 = 6$

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

You are framing a picture. The frames are available in 12 different styles. Each style is available in 55 colors. You also want a blue mat board, which is available in 11 shades of blue. How many different ways can you frame your picture?

$12 \cdot 55 \cdot 11 = 7260$ ways

Factorial: (!) n! = n (n-1)(n-2)....1.

Calculate: 6!

Permutations: An ORDERING of objects. The number of permutations of r objects taken from a group of n distinct objects is denoted ${}_nP_r$ and is given by : $${}_nP_r = \frac{n!}{(n-r)!}$$

$\frac{10!}{(10-6)!}$

${}_5P_3 = 60$

${}_{10}P_6 = 151,200$

The standard for a Texas license plate is 1 letter followed by 2 digits, followed by 3 letters.

a.) how many different plates are available?

$$26 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26$$
$$= 45,697,600$$

b.) how many possibilities if letters and digits cannot be repeated?

$$26 \cdot 10 \cdot 9 \cdot 25 \cdot 24 \cdot 23$$
$$= 32,292,000$$

If you want to burn a CD for a demo. You have 12 songs on your playlist. If you only want to put 4 songs on the demo CD, In how many orders can you burn 4 of the 12 songs?

$$_{12}P_4 = \frac{12!}{(12-4)!} = 11,880$$

Permutations with repeated elements: $\frac{n!}{p!q!}$

where n is the number of objects, p and q are # of repetitions of an object

How many words can you make from California?

#ofletters: 10

L: 2

a: 2

$10!$ ↓ non distinguishable

$\frac{10!}{2!2!}$ ↓ distinguishable

907,200

Find the number of distinguishable permutations of the letters in

a.) Parallel

# of letters: 8

a: 2

l: 3

$$\frac{8!}{2!3!} = 3{,}360$$

b.) TALLAHASSEE

# of letters: 11

a: 3

l: 2

s: 2

e: 2

$$\frac{11!}{3!2!2!2!} = 831{,}600$$

Combinations

Combinations vs. Permutations

order isn't important
(committee)

order is important
(presidency)

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_nP_r = \frac{n!}{(n-r)!}$$

How many 5 card hands of cards are possible?

$_{52}C_5 = 2{,}598{,}960$

Decide if it is a permutation or combination

a) A president, vice-president and secretary are chosen from a 25-member club.

P

b) A cook chooses 5 potatoes from a bag of 12 potatoes to make a potato salad.

C

c) Miss Mendel makes a seating chart for 22 students in a classroom with 30 desks.

P

A Pizzaria has 10 toppings for their pizzas. How many different pizzas can be formed if we can choose any 3 different toppings?

$$_{10}C_3 = \frac{10!}{3!(10-3)!} = 120$$

Counting Subsets of an n-Set:

There are 2^n subsets of a set with n objects (including the empty set and the entire set).

A hamburger chain offers 5 toppings for their burgers. If you want at least 3 toppings on your burger, how many possible choices are there?

Option 1

Option 2

9.3 Probability

Sample Space — set of all possible outcomes

Event — a subset of the sample space

Probability measures the likelihood of an event occurring. The desired outcome is called a success, any other outcome is a failure.

If an event cannot fail, its probability = 1 (guaranteed to happen). If it cannot succeed, the probability is 0(will never happen).

3 Types of Probability

Theoretical probability: (what happens in theory)

If an event can succeed in "s" ways and fail in "f" ways, then the probability of success when all outcomes are equally likely, the probabilty that event A will occur is:

$$P(A) = \frac{s}{s+f}$$

$s + f$ = number of total outcomes

You roll a die. What is the probability of getting a 4?

$\frac{1}{6}$

What is the probability of getting an odd number?

$\frac{3}{6}$ $\frac{1}{2}$

From a standard deck of cards, a card is drawn at random. Find the probability of drawing; 12/52

a.) a king $\frac{4}{52}$ b.) 4, 5 or 6

Color	Red	Green	Orange	Yellow
Proportion	.4	.2	.1	.3

If you draw disks from a bag, given the previous proportions, with replacement, what are the following probabilities?

What is the probability of drawing 2 reds?

$(.4)(.4) = .16$

What is the probability of drawing first orange then yellow?

$(.1)(.3) = .03$

What is the probability of drawing a red and a green?

$(.4)(.2) = .08$

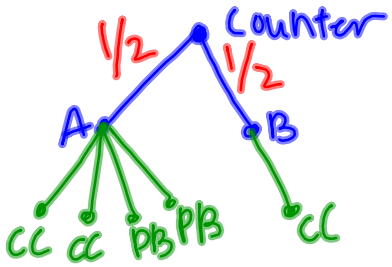
The ODDS of the successful outcome can be expressed as the ratio of the number of ways to succeed to the number of ways to fail or visa versa.

The odds in favor = # successes : # failures

What are the odds of drawing a queen from a deck of cards?

4 : 44

Conditional Probability



If the event B depends on the event A, then

$$P(B \mid A) = \frac{P(A \,\&\, B)}{P(A)}$$

two identical cookies jars are on a counter. Jar A contains 2 chocolate chip cookies and 2 peanut butter cookies, while jar B contains 1 chocolate chip cookie. We draw a cookie at random, if it is chocolate chip what is the probability it came from jar A?

$$\frac{(1/2)(2/4)}{(1/2)(1/4)} = \frac{1}{3}$$

Bracket Probability

What is the probability of getting a perfect bracket?

$$\frac{2^{63}}{9,223,372,036,854,775,808} \qquad \frac{1}{}$$