9-3 Operations with Complex Numbers

$$\sqrt{-1} = i \qquad \left(\sqrt{-1}\right)^2 = -1$$

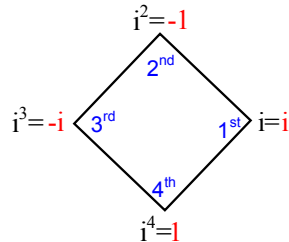**Powers of i**

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^2 = -1$ — 2nd

$i^3 = -i$ — 3rd   1st — i = i

4th

$i^4 = 1$

Pattern repeats every 4 iterations

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add/subtract: combine like terms (real w/real and imaginary w/imaginary)

Multiply: FOIL

Divide: factor or use conjugates to simplify (remember i is a radical and can't stay in the denominator)

Add:

$$(4 - 3i) + (-2 + 5i)$$

$$2 + 2i$$

$$\left(4 + \sqrt{-25}\right) + \left(-6 - \sqrt{-16}\right)$$

5i   4i

$$-2 + i$$

Subtract:

$$(-3 + 7i) - (5 - 4i)$$

$$-8 + 11i$$

$$i\sqrt{-12} \qquad \qquad (i)\sqrt{-27}$$

$$\left(3 + \sqrt{-12}\right) - \left(-2 - \sqrt{-27}\right)$$

$$\left(3 + 2i\sqrt{3}\right) - \left(-2 - 3i\sqrt{3}\right)$$

$$5 + 5i\sqrt{3} \qquad \qquad \sqrt{3} + 2\sqrt{3}$$

$$\qquad \qquad \qquad 3\sqrt{3}$$

$$3x + 2y \qquad \qquad x + 2x$$

$$\qquad \qquad \qquad 3x$$

You Try

$$\left(4 - \overset{2i}{\sqrt{-4}}\right) + \left(-7 + \overset{3i}{\sqrt{-9}}\right)$$

$$-3 + 1i$$

$$(4 - 2i) - (-2 + 7i)$$

$$6 - 9i$$

Multiply

$$4i(3 - 6i)$$

$$12i - 24i^2$$

$$12i - 24(-1)$$

$$\boxed{12i + 24}$$

$$(-2 + 4i)(3 - i)$$

$$-6 + 2i + 12i - 4i^2$$

$$-6 + 2i + 12i + 4$$

$$\boxed{-2 + 14i}$$

Remember from before:

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$

only works when $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers

DOES NOT WORK when there is a negative

This means that

$$\sqrt{a}\,\sqrt{b} \neq \sqrt{ab} \text{ if } a < 0 \text{ or } b < 0$$

Multiply

$$\sqrt{-25}\,\sqrt{-4} = \cancel{\sqrt{100}} = \cancel{10}$$

$$5i \cdot 2i$$

$$= 10i^2$$

$$= \boxed{-10}$$

$$(2 + \cancel{\sqrt{-16}})(1 - \cancel{\sqrt{-4}})$$

$$(2 + 4i)(1 - 2i)$$

$$2 - 4i + 4i + 8$$

$$\boxed{10}$$

You Try

$$\overset{3i}{\sqrt{-9}}\,\overset{6i}{\sqrt{-36}}$$

$3i \cdot 6i$
$18i^2$
-18
$6i$

$$\left(2+\sqrt{-36}\right)\left(4-\overset{5i}{\sqrt{-25}}\right)$$

$(2+6i)(4-5i)$

$8-10i+24i+30$

$38+14i$

Evaluate

i^1 i i^5 i

i^2 -1 i^6 -1

i^3 $-i$ i^7 $-i$

i^4 1 i^8 1

$(3+2i)^2=(3+2i)(3+2i)$