9-2 Complex Numbers... A history

European mathematicians were extremely weary of using negative numbers. In fact even in the 16th century some of the most famous mathematicians still rejected negative numbers as "fictitious" or "absurd".

Whenever those negative number appeared as solutions they considered the solution to be "fictitious" or "false roots".

But, by the early 17th century the tide was beginning to turn as negative numbers became too important and obvious to avoid.

Mathematicians still had many misunderstandings and were skeptical concerning negative quantities. The more they learned about solving equations the more confusion they encountered.

What happens when you want to solve $x^2 + 16 = 0$? Faced with this apparent absurdity Descartes (a very famous mathematician) called negative solutions "false" or "imaginary". According to Descartes the equation above only had one true root, 4, and one false root or imaginary, -4.

For these mathematicians there were NO NUMBERS in existence to solve these equations so they called these solutions, "false" or "imaginary".

In 1707 Sir Isaac Newton said "Quantities are either Affirmative, or greater than nothing, or Negative, or less than nothing." This definition was taken very seriously, but how could there be something less than nothing? It wasn't until the mid 18th century that negative number became accepted among mathematicians.

Even with the acceptance of negative numbers the idea of negative roots was still thought to be "impossible" "imaginary" and "useless"

In the middle of the 18th century Euler noted that, "All such expressions as $\sqrt{-1}, \sqrt{-2}, \sqrt{-3}, ..etc$ are impossible, or imaginary numbers, and of such numbers we may truly assert that they are neither nothing, nor greater than nothing, nor less than nothing; which necessarily constitutes them as imaginary or impossible.

But notwithstanding this, these number present themselves to the mind; they exist in our imagination, and we still have a sufficient idea of them."

It wasn't until 1831 that Gauss coined the term Complex number and showed how it could be useful mathematically. A few years later the complex number systems was created. These numbers turned out to be very powerful because it proved make calculus **easier.** Through the next hundred years complex numbers proved to be more useful in applied math that anyone could have imagined.

It is said that "the shortest path between two truths in the real domain passes through the complex domain". Even if we care only about real number problems and real number answers, the easiest solutions often involve complex numbers.

So why should we "believe" in complex numbers? Because they are so useful!

Complex Number System

standard form of a complex number

$\boxed{a} + \boxed{bi}$

real #'s
$b = 0$

Imaginary #'s

pure imaginary
$a = 0$

non-pure
$a \neq 0$

If
b=0 the # is real
b≠0 the # is imaginary
a = 0 the # is pure imaginary

Def of Equal Complex #'s: a+bi = c+di iff a =c and b = d
so 2 complex #'s are equal if the real parts are equal & imaginary parts are equal

What are the real and imaginary components

$-2 + 4i$
R: -2 i: $4i$

$-7i$
R: 0 i: $-7i$

Find x and y:

$3 + 5i = x + yi$
$x = 3$
$y = 5$

$x + 5i = (2 - i) + (4 - 2yi)$

Complex Numbers

$\sqrt{-1} = i$ $\quad (\sqrt{-1})^2 = -1$

$i^3 = i \cdot i^2$
$i^3 = i \cdot i \cdot i$

Powers of i

$i = \sqrt{-1}$

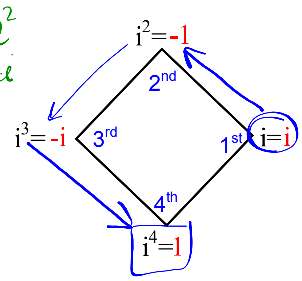
$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^4 = i \cdot i \cdot i \cdot i$
$= i^2 \cdot i^2$
$= (-1)(-1)$
$= 1$

$i^8 = i^4 \cdot i^4$
$1 \cdot 1 = 1$



Pattern repeats every 4 iterations

Simplify:

1. $\sqrt{-81}$

$\pm 9i$

2. $\sqrt{-121x^5}$

$x^4 \cdot x^1$

$\pm 11x^2 i\sqrt{x}$

3. $8i \bullet 3i$

$24i^2$

$\boxed{-24}$

4. $\sqrt{-5} \bullet \sqrt{-20}$

$\sqrt{100} = \pm 10$

Simplify:

1. $i^{12} = 1$

$i^{12} = i^4 i^4 i^4$

$= 1 \cdot 1 \cdot 1$

2. $i^{43} = i^{40} \cdot i^3$

$= 1 \cdot -i$

$= -i$

3. $i^{81} = i^{80} \cdot i^1$

$= 1 \cdot i = i$

4. $i^{60} = 1$

Write the following in standard form. State the real and imaginary parts.

$7 + \sqrt{-36}$

$7 \pm 6i$

R I

$\dfrac{6 - \sqrt{-8}}{12}$

8 = 2 · 4 = 2 · 2 · 2

$\dfrac{6 - 2\sqrt{-2}}{12}$

$\dfrac{6 - 2i\sqrt{2}}{12}$

R $\dfrac{3 - i\sqrt{2}}{6}$ i

R: $\frac{3}{6}$

I: $\frac{-i\sqrt{2}}{6}$

Solve the following Quadratics:

$x^2 + 49 = 0$

$-49 \quad -49$

$\sqrt{x^2} = \sqrt{-49}$

$x = \pm 7i$

$3x^2 + 75 = 0$

$-75 \quad -75$

$\frac{3x^2}{3} = \frac{-75}{3}$

$\sqrt{x^2} = \sqrt{-25}$

$x = \pm 5i$

$x^2 - 121 = 0$

$+121 \quad +121$

$\sqrt{x^2} = \sqrt{121} \quad x = \pm 11$

$(x-4)^2 + 100 = 0$

$-100 \quad -100$

$\sqrt{(x-4)^2} = \sqrt{-100}$

$x - 4 = \pm 10i$

$+4 \quad +4$

$x = 4 \pm 10i$

Solve the following Quadratics:

$x^2 + 80 = 0$

$-80 \quad -80$

$\sqrt{x^2} = \sqrt{-80}$

$x = \pm\sqrt{-80}$

$x = \pm 4i\sqrt{5}$

$2(x+3)^2 + 50 = 0$

-50

$\frac{2(x+3)^2}{2} = \frac{-50}{2}$

$\sqrt{(x+3)^2} = \sqrt{-25}$

$x+3 = \pm 5i$

$-3 \quad -3$

$x = -3 \pm 5i$

$3(x-7)^2 + 36 = 0$

$-36 \quad -36$

$\frac{3(x-7)^2}{3} = \frac{-36}{3}$

$\sqrt{(x-7)^2} = \sqrt{-12}$

$x - 7 = \pm\sqrt{-12}$

$x - 7 = \pm 2i\sqrt{3}$

$+7 \quad +7$

$x = 7 \pm 2i\sqrt{3}$

$6(x-7)^2 - 60 = 0$

$+60 \quad +60$

$\frac{6(x-7)^2}{6} = \frac{60}{6}$

$\sqrt{(x-7)^2} = \sqrt{10}$

$x - 7 = \pm\sqrt{10}$

$+7 \quad +7$

$x = 7 \pm \sqrt{10}$