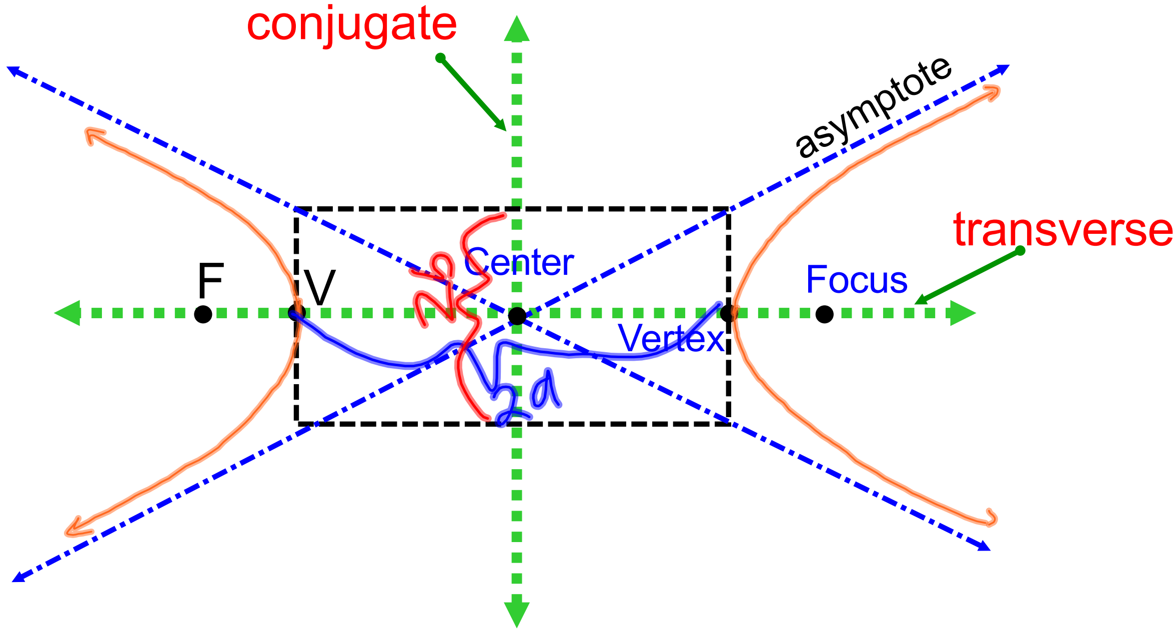
8.3 Hyperbolas

#86

hyperbola: a set of all points in a plane whose distances from two fixed points(foci) in the plane have a constant difference.



focal axis - line through the foci

center - midpoint of the seg. connecting foci or vertices

vertices - points where hyperbola intersects the focal axis

asymptotes - the 2 guidelines the hyperbola approaches but never crosses

transverse axis - a line segment 2a units long whose endpts lie on the vertices (through the foci)

conjugate axis - line segment 2b units long that is $\perp$ to the transverse axis

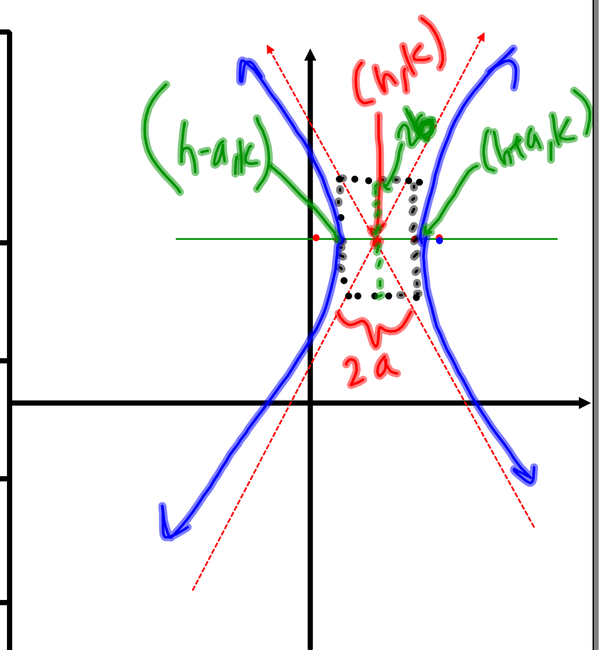
pythagorean relationship: $\boxed{c^2 = a^2 + b^2}$

Hyperbola - Standard Form

#87

horizontal

Standard Eq	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Center	(h, k)
Foci	$(h \pm c, k)$
Vertices	$(h \pm a, k)$
Asymptotes	$y = \pm\frac{b}{a}(x-h)+k$
Pythagorean Relationship	$a^2 + b^2 = c^2$

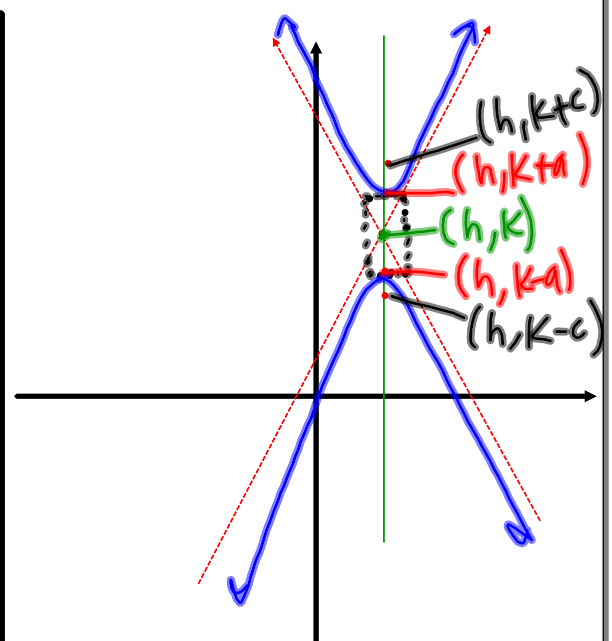


Hyperbola - Standard Form

#87 - back

vertical

Standard Eq	$\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$
Center	(h, k)
Foci	$(h,k\pm c)$
Vertices	$(h,k\pm a)$
Asymptotes	$y=\pm\dfrac{a}{b}(x-h)+k$
Pythagorean Relationship	$a^2+b^2=c^2$



Find the center, vertices and foci of the hyperbola

$$\frac{x^2}{16} - \frac{y^2}{7} = 1$$

C: (0,0)

horizontal - change x coordinate

$a^2 = 16$ V: $(\pm 4, 0)$

$a = \pm 4$

$16 + 7 = c^2$ F: $(\pm\sqrt{23}, 0)$

$c = \pm\sqrt{23}$

Find the center, vertices, and foci. Sketch a graph.

$$\frac{x^2}{16} - \frac{y^2}{49} = 1$$

$C: (0,0)$

Transverse $2a$

$16 = a^2$

$a = \pm 4$

Conjugate $2b$

$49 = b^2$

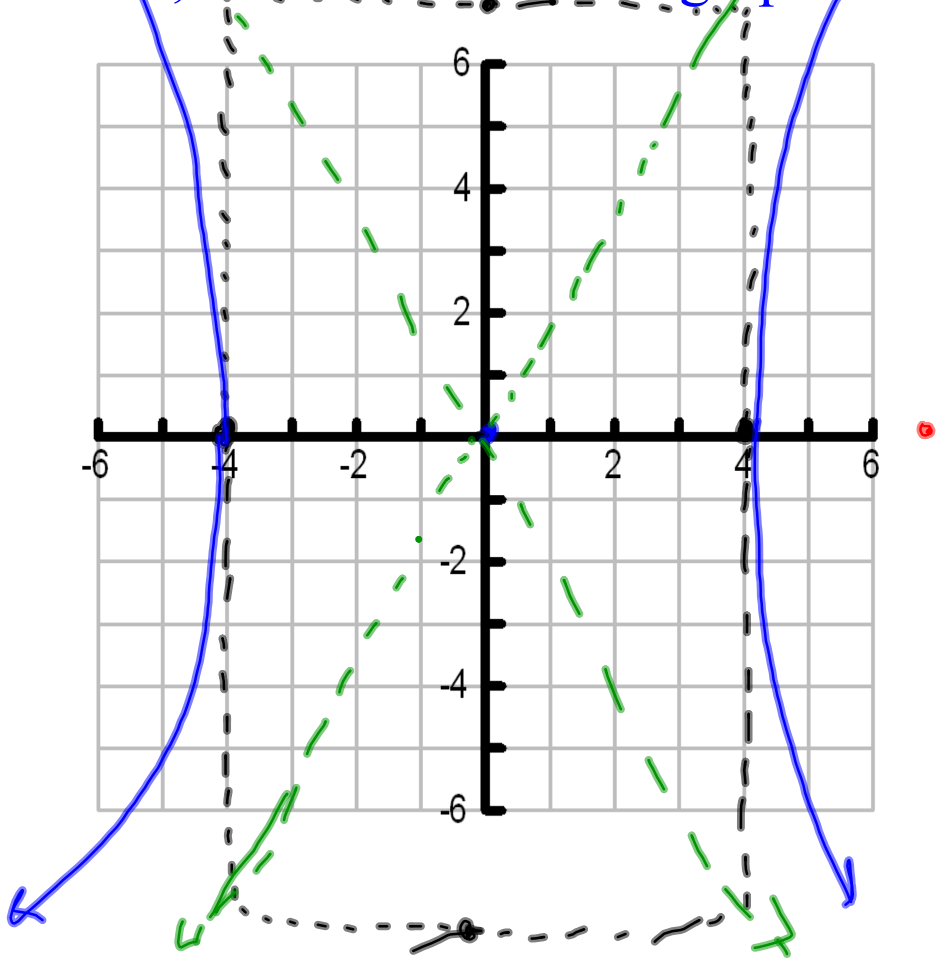
$b = 7$

$V: (\pm 4, 0)$

$F: (\pm\sqrt{65}, 0)$

$16 + 49 = c^2$

$c = \pm\sqrt{65}$



Find the center, vertices, and foci. Sketch a graph.

$$\frac{4(y-1)^2 - 9(x-3)^2}{36} = \frac{36}{36}$$

$$\frac{(y-1)^2}{9} - \frac{(x-3)^2}{4} = 1$$

C: $(3, 1)$

Transerve: $2a$

$9 = a^2$

$a = 3$

V: $(3, -2)$, $(3, 4)$

Conjugate: $2b$

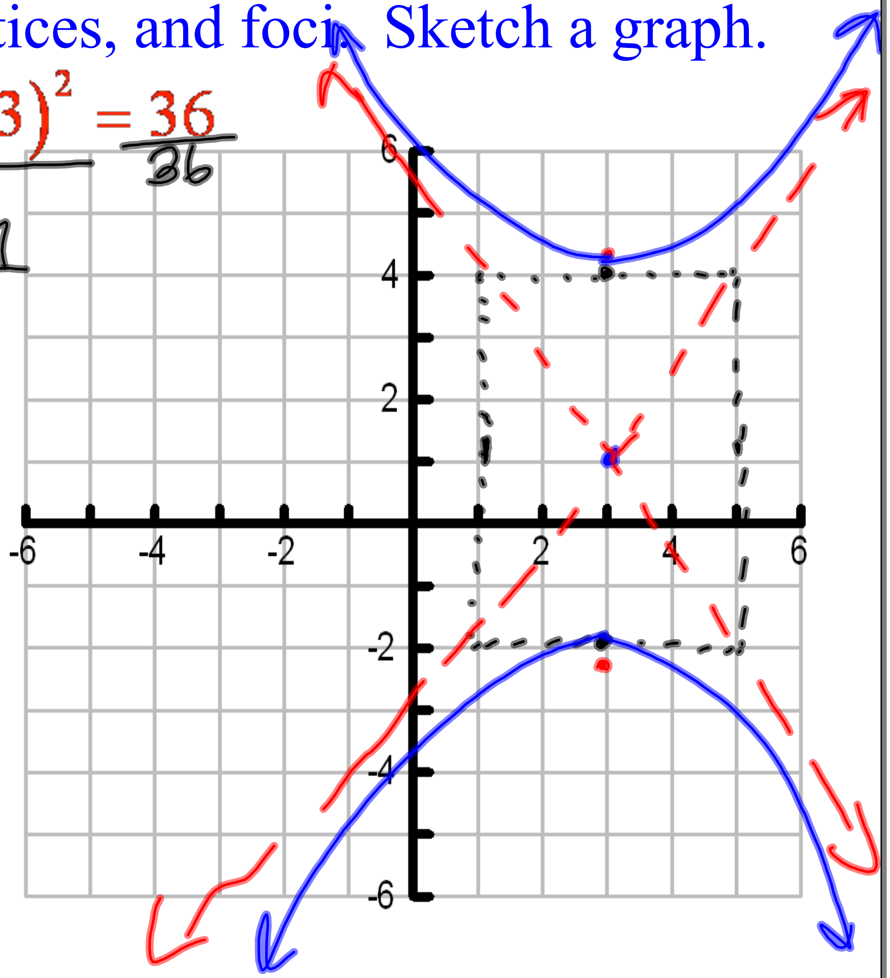
$b^2 = 4$

$b = \pm 2$

$9 + 4 = c^2$

$c = \pm\sqrt{13}$

F: $(3, 1 \pm \sqrt{13})$



Write the equation of the hyperbola:

foci: $(\pm 3,0)$ HORIZONTAL $c:(0,0)$

trans. axis length 4 $=2a$

$a=2$ $c=3$

$9=4+b^2$

$5=b^2$

$$\frac{x^2}{4}-\frac{y^2}{5}=1$$

Write the equation of the hyperbola: VERTICAL

trans axis endpts: (2, 3) and (2, -1)

conj. axis = 6 $= 2b$

$b = 3$

$4 = 2a$

$a = 2$

$C: (2, 1)$

$$\frac{(y-1)^2}{4} - \frac{(x-2)^2}{9} = 1$$

#88

Hyperbola - General Form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

when A is neg - vertical hyp. or C is neg - horizontal hyp.

Steps:

1. move variables to left & constants to right side of eq. to complete the square
2. Group like variables
3. If x^2 & x terms, complete sq. for x's
4. If y^2 & y terms, complete sq. for y's
5. Write each completed sq. in factored form.
6. Need to have 1 on rt. so divide both sides by value on rt.
7. Simplify
8. result is in graphing form

Write the equation of the hyperbola in standard form:

$$5x^2 - 4y^2 - 40x - 16y = 36$$

$$5x^2 - 40x - 4y^2 - 16y = 36$$

$$5(x^2 - 8x + \underline{16}) - 4(y^2 + 4y + \underline{4}) = 36 + 5(16) - 4(4)$$

$$\frac{-8}{2} = (-4)^2$$

$$\frac{4}{2} = (2)^2$$

$$\frac{5(x-4)^2 - 4(y+2)^2}{100} = \frac{100}{100}$$

$$\frac{(x-4)^2}{20} - \frac{(y+2)^2}{25} = 1$$