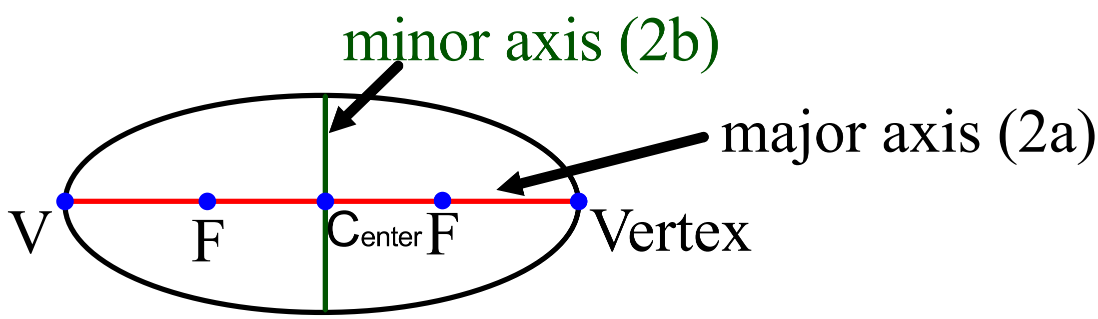
8.2 Ellipses

#83

ellipse: a set of all points in a plane whose distances from two fixed points (foci) in the plane have a constant sum.



foci - focus plural (always on major axis)

focal axis - line through the foci

center - midpoint of the foci (intersection of major & minor axes)

vertices - points where ellipse intersects the major axis

major axis - chord through the foci (longer)

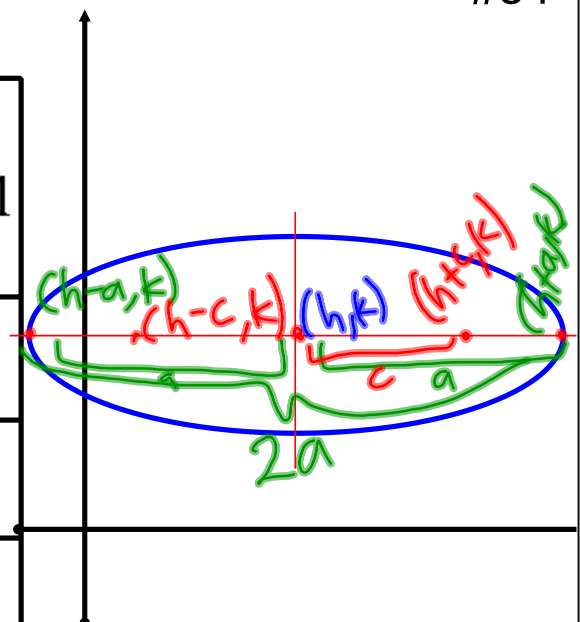
minor axis - chord through the center perpendicular to the major axis (shorter)

pythagorean relationship: $a^2 = b^2 + c^2$

Ellipse - Standard form
horizontal

#84

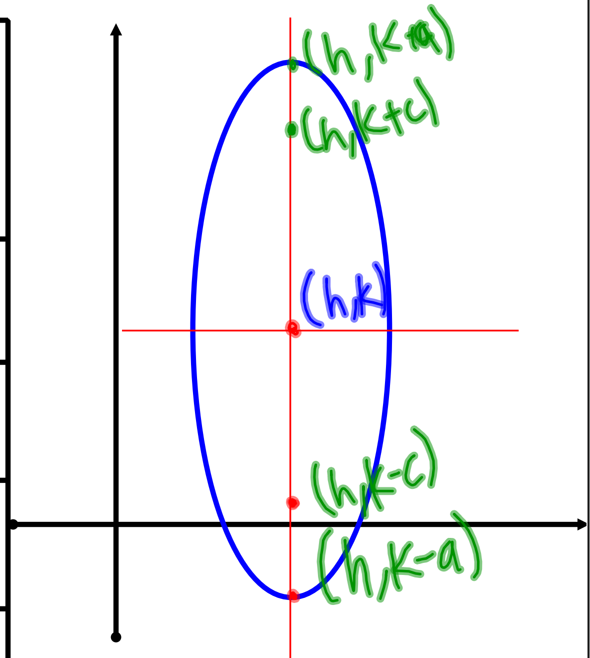
Standard Eq	$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$
Center	(h, k)
Foci	$(h \pm c, k)$
Vertices	$(h \pm a, k)$
Focal axis	y = k
Pythagorean Relationship	$a^2=b^2+c^2$



Ellipse - Standard form
vertical

#84-
back

Standard Eq	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Center	(h, k)
Foci	$(h, k \pm c)$
Vertices	$(h, k \pm a)$
Focal axis	x = h
Pythagorean Relationship	$a^2=b^2+c^2$



Find the vertices and foci of $\dfrac{4x^2 + 9y^2}{36} = \dfrac{36}{36}$

$C: (h, k)$

$V: (h \pm a, k)$

$F: (h \pm c, k)$

$$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$C: (0, 0)$

$V: (0 \pm 3, 0)$

$(\pm 3, 0)$

$F: (0 \pm \sqrt{5}, 0)$

$(\pm \sqrt{5}, 0)$

$a^2 = 9$

$a = \pm 3$

$a^2 = b^2 + c^2$

$9 = 4 + c^2$

$5 = c^2$

$c = \pm \sqrt{5}$

Find the vertices and foci of $$\frac{x^2}{16}+\frac{y^2}{7}=1$$

C: (0,0)

a: ±4

v = (±4, 0)

F: (0 ± 3, 0)

$16 = 7 + c^2$

-7

$\sqrt{9} = c^2$

$c = \pm 3$

Find the center, vertices, and foci. Sketch a graph.

$a^2=25 \Rightarrow a=\pm 5$ $25=16+c^2$ $9=c^2 \Rightarrow c=\pm 3$

$$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{16}=1$$

C: (2,-1)

V: $(2\pm 5,-1)$

(7,-1)(-3,-1)

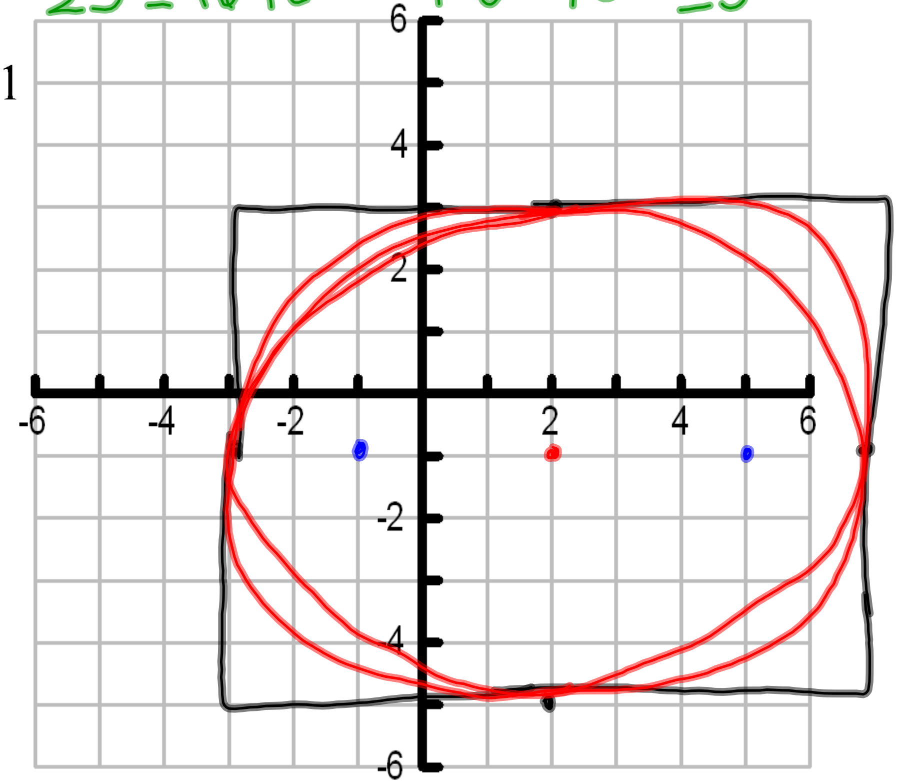
F: $(2\pm 3,-1)$

(5,-1)(-1,-1)

minor axis: 2b

$16=b^2$ $2b=8$

$b=4$



Find the center, vertices, and foci. Sketch a graph.

$a^2=9 \quad a=\pm 3 \qquad 9=4+c^2 \Rightarrow c^2=5 \Rightarrow c=\pm\sqrt{5}$

$$\frac{9(x-2)^2+4(y+3)^2}{36}=\frac{36}{36}$$

$$\frac{(x-2)^2}{4}+\frac{(y+3)^2}{9}=1$$

$(2,-3)$

V: $(2,-3\pm 3)$

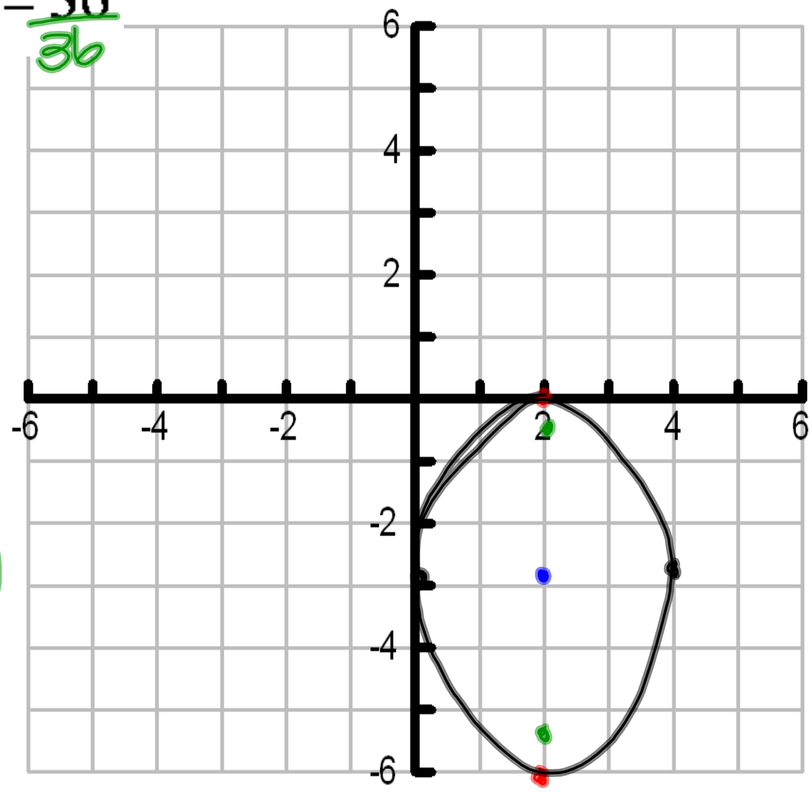
$(2,0)\ (2,-6)$

F: $(2,-3\pm\sqrt{5})$

$2b=4$

$b^2=4$

$b=\pm 2$



Write the equation of the ellipse:

Major axis endpts: $(\pm 5, 0)$ $10 = 2a$ $C: (0,0)$

minor axis length 4 $= 2b$ $a = 5$

$b = 2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Write the equation of the ellipse:

foci: (1, -4) and (5, -4)

major axis endpts: (0, -4) and (6, -4)

H (, -4)

$6 = 2a$

$a = 3$

$5 = h + c$

$5 = 3 + c$

$2 = c$

$a^2 = b^2 + c^2$

$9 = b^2 + 4$

$b^2 = 5$

$(h + a, k)$

$(h - a, k)$

$6 = h + 3$

$h = 3$

$$\frac{(x-3)^2}{9} + \frac{(y+4)^2}{5} = 1$$

Ellipse - General Form

#85

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Steps:

1. move variables to left & constants to right side of eq. to complete the square
2. Group like variables
3. If x^2 & x terms, complete sq. for x's
4. If y^2 & y terms, complete sq. for y's
5. Write each sq. in factored form.
6. Need to have 1 on rt. so divide both sides by value on rt.
7. Simplify
8. result is in graphing form

Write the equation of the ellipse in standard form:

$$9x^2 + 16y^2 + 54x - 32y - 47 = 0$$
$$+47 \quad +47$$

$$9x^2 + 54x + 16y^2 - 32y = 47$$

$$9(x^2 + 6x + \underline{9}) + 16(y^2 - 2y + \underline{1}) = 47 + 9(9) + 16(1)$$

$$\frac{9(x+3)^2 + 16(y-1)^2}{144} = \frac{144}{144}$$

$$\frac{(x+3)^2}{16} + \frac{(y-1)^2}{9} = 1$$