8.2 Ellipses

ellipse: a set of all points in a plane whose distances from two fixed points (foci) in the plane have a constant sum.
**foci** - focus plural (always on major axis)

**focal axis** - line through the foci

**center** - midpoint of the foci (intersection of major & minor axes)

**vertices** - points where ellipse intersects the major axis

**major axis** - chord through the foci (longer)

**minor axis** - chord through the center perpendicular to the major axis (shorter)

Pythagorean relationship: \[ a^2 = b^2 + c^2 \]
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<th>Ellipse - Standard form</th>
<th>horizontal</th>
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<td><strong>Standard Eq</strong></td>
<td>[ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 ]</td>
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<td><strong>Center</strong></td>
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<td><strong>Foci</strong></td>
<td>(h ± c, k)</td>
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Find the vertices and foci of \[ 4x^2 + 9y^2 = \frac{36}{36} \]

\[ \frac{4x^2}{36} + \frac{9y^2}{36} = 1 \]

\[ \frac{x^2}{9} + \frac{y^2}{4} = 1 \]

\[ a^2 = 9 \quad a = \pm 3 \]

\[ b^2 = 4 \quad b = \pm 2 \]

\[ c^2 = 5 - c^2 \quad c = \pm \sqrt{5} \]

C: \((0,0)\)
V: \((\pm 3,0)\) \((\pm \sqrt{5},0)\)
F: \((\pm \sqrt{5},0)\)
Find the vertices and foci of \[
\frac{x^2}{16} + \frac{y^2}{7} = 1
\]

C \((0,0)\)

\(a = \pm 4\)

\(v = (\pm 4, 0)\)

\(F = (0, \pm 3)\)

\[16 = 7 + c^2\]

\[\sqrt{9} = c^2 \quad c = \pm 3\]
Find the center, vertices, and foci. Sketch a graph.

$$a^2 = 25 \Rightarrow a = \pm 5$$
$$\frac{(x - 2)^2}{25} + \frac{(y + 1)^2}{16} = 1$$

$$25 = 16 + c^2 \Rightarrow c^2 = 9 \Rightarrow c = \pm 3$$

Center \(C\): \((2, -1)\)

Vertices \(V\): \((2 \pm 5, -1)\) and \((-3, -1)\)

Foci \(F\): \((2 \pm 3, -1)\) and \((-1, -1)\)

Minor axis: \(2b\)

$$b^2 = 16 - 9 = 7$$

$$b = \sqrt{7}$$

$$2b = 2\sqrt{7} = 8$$
Find the center, vertices, and foci. Sketch a graph.

\[
\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1
\]

\[
\begin{align*}
(a^2, b^2) &= (9, 4) \\
a &= \pm 3 \\
b &= \pm 2 \\
a^2 + b^2 &= c^2 \
9 + 4 &= 5 \\
c &= \pm \sqrt{5}
\end{align*}
\]

Center: \((2, -3)\)

Vertices: \((2, -3 \pm 3)\)

Foci: \((2, -3 \pm \sqrt{5})\)
Write the equation of the ellipse:

Major axis endpts: \((\pm 5, 0)\)
minor axis length 4

\[10 = 2a \quad \Rightarrow \quad a = 5\]
\[= 2b \quad \Rightarrow \quad b = 2\]

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

\[
\frac{x^2}{25} + \frac{y^2}{4} = 1
\]
Write the equation of the ellipse:

- foci: (1, -4) and (5, -4)
- major axis endpts: (0, -4) and (6, -4)

\[ \begin{align*}
5 &= h + c \\
5 &= 3 + c \\
2 &= c \\
a &= \frac{b}{2} \\
a^2 &= b^2 + c^2 \\
b^2 &= 5 \\
\frac{(x - 3)^2}{9} + \frac{(y + 4)^2}{5} &= 1
\end{align*} \]
Ellipse - General Form

\[ Ax^2 + Cy^2 + Dx + Ey + F = 0 \]

Steps:

1. move variables to left & constants to right side of eq. to complete the square
2. Group like variables
3. If \( x^2 \) & x terms, complete sq. for x's
4. If \( y^2 \) & y terms, complete sq. for y's
5. Write each sq. in factored form.
6. Need to have 1 on rt. so divide both sides by value on rt.
7. Simplify
8. result is in graphing form
Write the equation of the ellipse in standard form:

$$9x^2 + 16y^2 + 54x - 32y - 47 = 0$$

$$+47 + 47$$

$$9(x^2 + 6x + 9) + 16(y^2 - 2y + 1) = 47 + 9(9) + 16(1)$$

$$9(x + 3)^2 + 16(y - 1)^2 = 144$$

$$\frac{(x+3)^2}{16} + \frac{(y-1)^2}{9} = 1$$