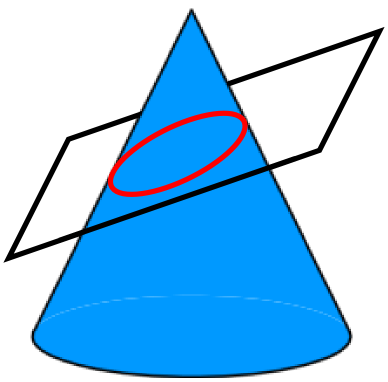
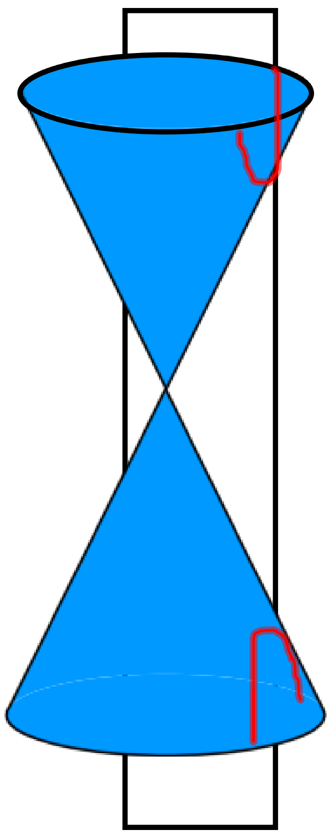
8.1 Conic Sections and Parabolas

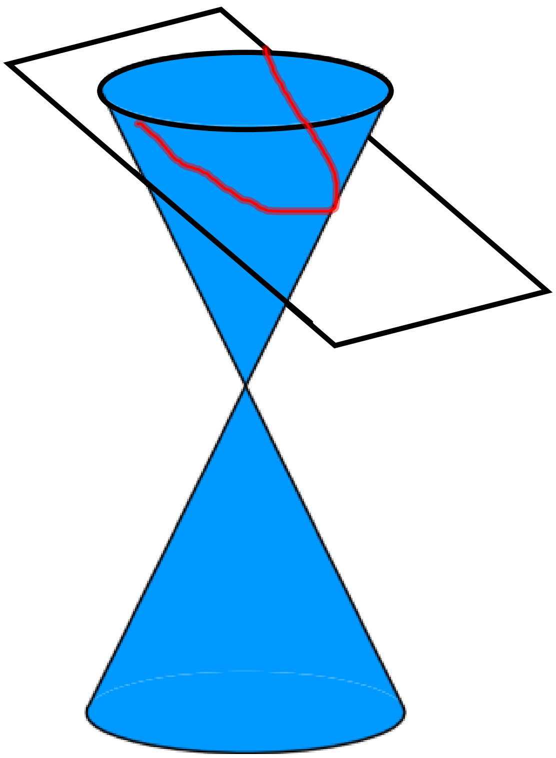


ellipse - intersects at angle

circle - the plane has to be parallel to the base of the cone

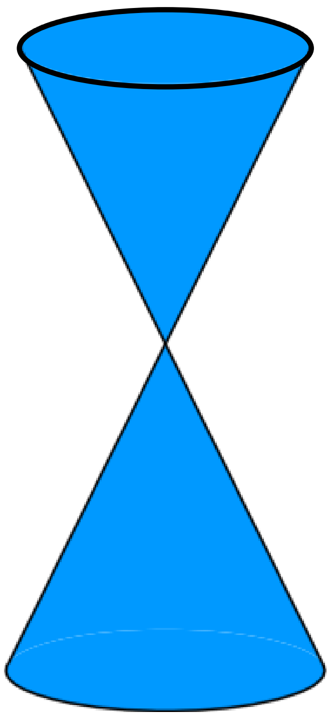


hyperbola



parabola

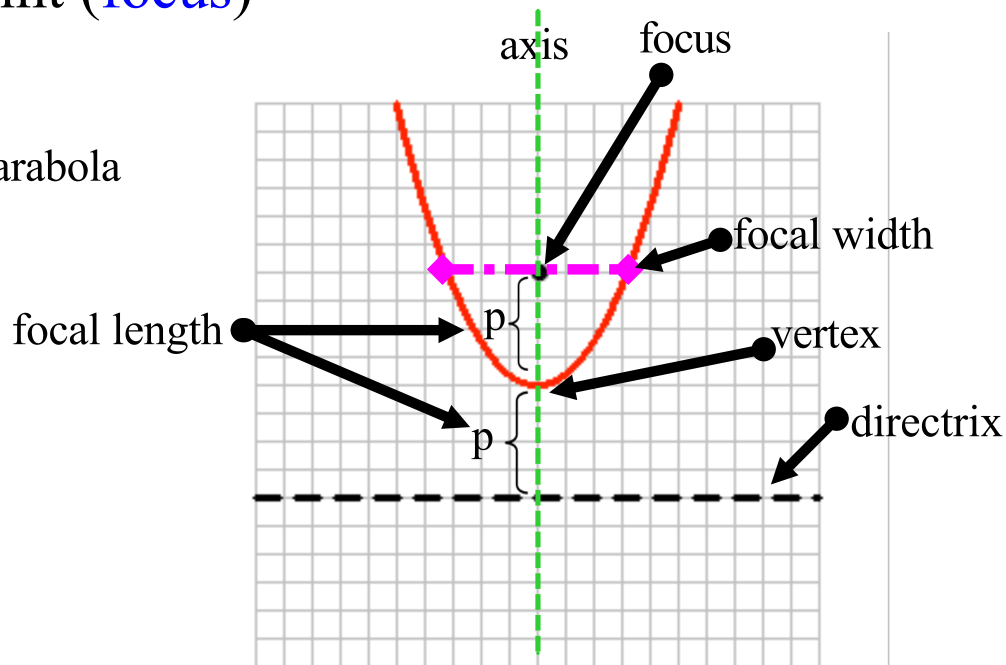
Degenerate forms



Point
Line
Intersecting Lines

Parabola : set of all points in a plane equidistant from a particular line (directrix) and a particular point (focus) #80

the parts of a parabola



axis of symmetry is $\perp$ to the directrix

Latus rectum = focal width - the segment thru the focus $\perp$ to the axis of symmetry. Its endpts lie on the parabola & length = $|4p|$ (parallel to the directrix)

Axis of Symmetry - line $\perp$ to the latus rectum & directrix. It intersects the parabola at the vertex.

measure from the focus to an endpt of the latus rectum = measure from the focus to the directrix.

Parabola - standard form

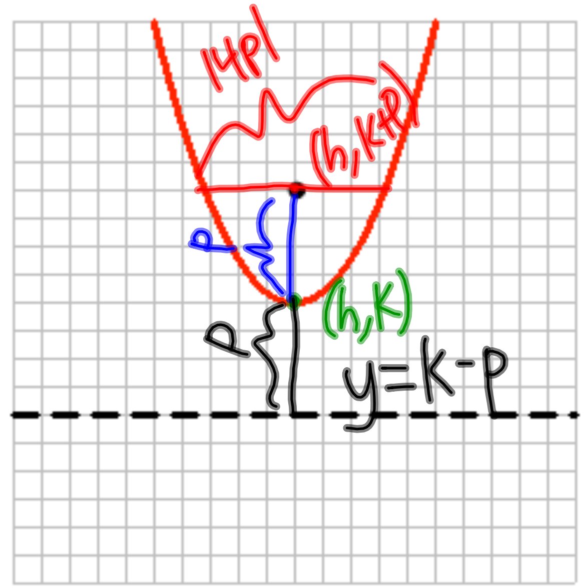
#81

up/down

$$\boxed{4p(y-k) = (x-h)^2}$$

vertical axis of symmetry

vertex	(h, k)
focus	(h, k + p)
directrix	y = k - p
axis	x = h
focal length	p
focal width	$\lvert 4p \rvert$



81 -
back

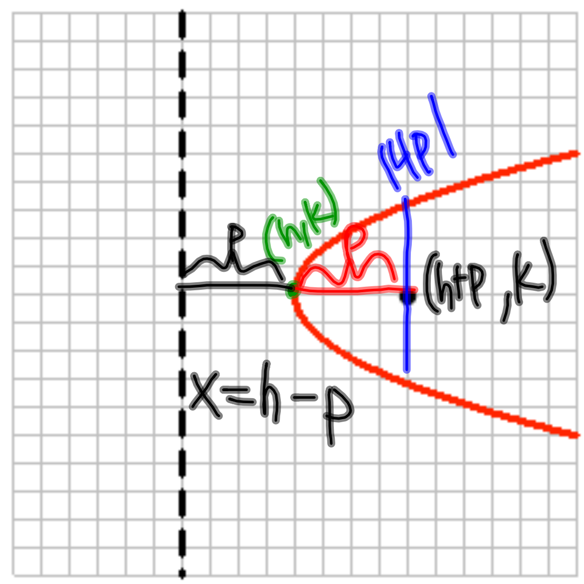
left/right

$$4p(x-h)=(y-k)^2$$

horizontal axis of symmetry

vertex	(h, k)
focus	(h + p, k)
directrix	x = h - p
axis	y = k
focal length	p
focal width	$\lvert 4p \rvert$

(not a function)



Graph: $2(x-2)=(y-3)^2$

Graph: Vertex, Focus, Directrix

2 focal width endpoints

$\frac{2}{4}=\frac{4p}{4}$ $p=\frac{1}{2}$

vertex $(2,3)$

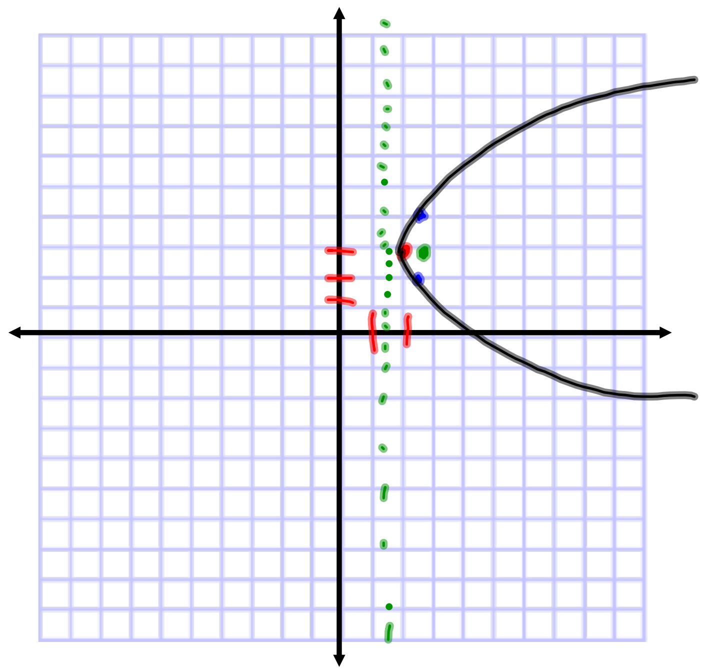
focus $(5/2, 3)$

directrix $x=3/2$

axis $y=3$

focal length $1/2$

focal width 2



Example:

Write the equation for a parabola with V: (2, -1) and a focal width of 4, opening down.

$$4p(y-k)=(x-h)^2$$

$$4p(y+1)=(x-2)^2$$

$$\boxed{-4(y+1)=(x-2)^2}$$

$$|4p|=4$$

$$\frac{4p}{4}=\frac{\pm 4}{4}$$

$$p=\pm 1$$

Write the equation for a parabola with V: (4, 3) and directrix x = 6

$$4p(x-h)=(y-k)^2$$

$$4p(x-4)=(y-3)^2$$

$x=h-p$

$6=4-p$

$-4 \quad -4$

$p=2$

$$-8(x-4)=(y-3)^2$$

#82

Parabola - General Form

$$Ax^2 + Dx + Ey + F = 0$$

$$Cy^2 + Dx + Ey + F = 0$$

Steps:

1. move the variable w/o a square term to the left & everything else to the rt.

2. Complete the sq. w/ the variables that have a sq. & linear term.

3. Write the completed square in factored form

4. Simplify

Prove the graph of the equation is a parabola, find the vertex, focus and directrix

$$y^2 - 3x + 6y + 12 = 0$$

$+3x \quad -12$

$$\underbrace{y^2 + 6y}_{\text{complete square}} = 3x - 12$$

$\left(\frac{6}{2}\right)^2$

$\left(\frac{6}{2}\right) = 3$

$$y^2 + 6y + 9 = 3x - 12 + 9$$

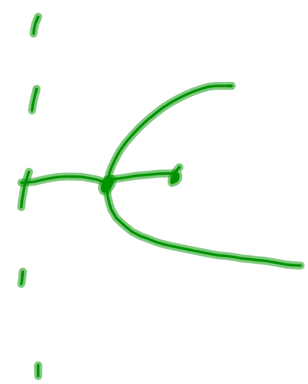
$$(y+3)^2 = 3x - 3$$

$$(y+3)^2 = 3(x-1)$$

$3 = 4p$

$\frac{3}{4} = \frac{4p}{4}$

$p = \frac{3}{4}$



V: $(1, -3)$

F: $\left(\frac{7}{4}, -3\right)$

d: $x = 1/4$

Prove the graph of the equation is a parabola, find the vertex, focus and directrix

$$3x^2 - 6x - 6y + 10 = 0$$

$+6y \quad -10$

$3x^2 - 6x = 6y - 10$

$\frac{-2}{2} = -1$

$3(x^2 - 2x + 1) = 6y - 10 + 3$

$3(x-1)^2 = 6y - 7$

$\frac{3(x-1)^2}{3} = \frac{6(y - 7/6)}{3}$

$4p = 2$

$p = 1/2$

$(x-1)^2 = 2(y - 7/6)$

$V: (1, 7/6) \quad F: (1, 5/3) \quad d: y = 2/3$