7.4 Partial Fractions

In 2.6 we learned how to break down a polynomial of higher degree using our factoring skills and synthetic division.

IE: Factor: \( x^2 + 3x + 2 = (x+1)(x+2) \)

Breaking complicated expressions into less complicated expressions is a mathematician's most important tool. Today we are going to break down rational functions.

Write the sum as a single rational function

\[
\frac{3(x+3)}{(x+3)(x-4)} \quad \frac{2(x-4)}{x+3} \quad \frac{3(43)}{(x+3)(x-4)} \quad \frac{2(x-4)}{(x+3)(x-4)}
\]

\[
\frac{3x+9}{x^2-x-12} + \frac{2x-8}{x^2-x-12} = \frac{5x+1}{x^2-x-12}
\]
Rational Functions

Unlike polynomials which break down into a product of factors, rational functions break down into a sum of fractions. This is called Partial Fraction Decomposition

\[
\frac{3x - 4}{x^2 - 2x} = \frac{2}{x} + \frac{1}{x - 2}
\]

Each fraction in the sum is called a partial fraction the entire sum is the partial fraction decomposition.

Find the partial fraction decomposition of:

\[
\frac{(x+3)(x-5)}{5x-1} = \frac{A}{x-5} + \frac{B}{x+3}
\]

\[
\frac{Sx - 1}{(x+3)(x-5)} = A\frac{x+3}{x+3} + B\frac{x-5}{x-5}
\]

\[
Sx - 1 = A(x+3) + B(x-5)
\]

\[
5x - 1 = 6A - 8B + 3B
\]

\[
Sx - 1 = Ax + 3A - 8B + B
\]

\[
\begin{align*}
A + B &= 5 \\
-8A + 3B &= -1
\end{align*}
\]

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} = \begin{bmatrix}
1 & 1 \\
-8 & 3
\end{bmatrix}^{-1} \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
\frac{1}{5} \\
\frac{3}{5}
\end{bmatrix}
\]

\[
\frac{2}{x+3} + \frac{3}{x-5}
\]
Find the partial fraction decomposition of:

\[
\frac{3(x-1)}{x^2 + 5x + 4} = \frac{A}{x+4} + \frac{B}{x+1}
\]

\[
3(x-1) = A(x+1) + B(x+4)
\]

\[
3x - 3 = Ax + A + Bx + 4B
\]

\[
3x - 3 = (A + B)x + (A + 4B)
\]

\[
A + B = 3
\]

\[
A + 4B = -3
\]

\[
\begin{bmatrix} 1 & 1 \\ 4 & 1 \\ -3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 3 \\ 4 \\ -1 \end{bmatrix}
\]

\[
\frac{S}{x+4} + \frac{-2}{x+1}
\]

Find the partial fraction decomposition of:

\[
\frac{7}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}
\]

\[
7 = A(x-3) + B(x-2)
\]

\[
7 = Ax - 3A + Bx - 2B
\]

\[
7 = (A + B)x + (-3A - 2B)
\]

\[
A + B = 0
\]

\[
-3A - 2B = 7
\]

\[
\begin{bmatrix} 1 & 1 \\ 4 & 1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}
\]

\[
\begin{bmatrix} -7 \\ 7 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \end{bmatrix}
\]

\[
\frac{-7}{x-2} + \frac{7}{x-3}
\]