Review of Inverses

A matrix has an inverse if:

- It is Square
- The det(A)≠0

If the det(A)=0 then the matrix is SINGULAR

The determinant of a 2x2 matrix is ad-bc $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Inverse of a 2x2 Matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Determinant

$A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$

$3 \cdot 2 - 4 \cdot 1$

$6 - 4$

$\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix} = A^{-1}$

Inverse of a 3x3 Matrix

Plug into your calculator

$$\begin{pmatrix} 3 & -3 & 6 \\ 1 & -3 & 10 \\ -1 & 3 & -5 \end{pmatrix}$$

Matrices with Inverses

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Solving with inverses:

$$x - 3y + z = 4$$

$$-y - 4z = 7$$

$$5x - 13y + 13z = 8$$

coefficient matrix

$$\begin{pmatrix} 1 & -3 & 1 \\ 0 & -1 & -4 \\ 5 & -13 & 13 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

variable matrix

answer matrix

$$\begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}$$

coefficient matrix

3x3

$$\begin{pmatrix} 1 & -3 & 1 \\ 0 & -1 & -4 \\ 5 & -13 & 13 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}$$

answer matrix

3x1

variable matrix

C^{-1} C^{-1}

$C \cdot V = A$

$C^{-1} \cdot C \cdot V = C^{-1} A$

$I \cdot V = C^{-1} A$

$V = C^{-1} A$

to solve:

you have to have the same number of variables as equations to use this method

possible answers:

you obtain solutions

singular matrix - no solutions
(because there was no inverse for C)

2x2 Matricies: BY HAND

$$3x - 2y = 0$$

$$-x + y = 5$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$C \quad V = A$

$V = C^{-1}A$

$$\frac{1}{3-2}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$(10, 15)$

Solve using inverse matrices:

$$2x - y + z = -6$$

$$x + 2y - 3z = 9$$

$$3x - 2y + z = -3$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 9 \\ -3 \end{bmatrix}$$