7.3 Matrices and Systems

Solve the system:

$$x - 2y + z = 7 \quad)-3$$
$$-3x + 6y - 3z = -21$$
$$3x - 5y + z = 14$$
$$-2x + 4y - 2z = -14$$
$$2x - 2y - z = 3$$

$$x - 2y + z = 7$$
$$-2\,(y - 2z = -7)$$
$$-2y + 4z = 14$$
$$2y - 3z = -11$$

$$x - 2y + z = 7$$
$$y - 2z = -7$$
$$\boxed{z = 3}$$

$$y - 2(3) = -7$$
$$y - 6 = -7$$
$$\boxed{y = -1}$$

$$x - 2(-1) + 3 = 7$$
$$x + 2 + 3 = 7$$
$$x + 5 = 7$$
$$\boxed{x = 2}$$

$$(2, -1, 3)$$

That wasn't fun!!

$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

Instead lets use matrices
to record the variables
and use row operations
to solve

Systems with Matrices

$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

augmented matrix

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array}\right) \end{array} \quad \text{or} \quad \begin{pmatrix} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{pmatrix}$$

COEFFICIENT matrix

Write the augmented matrix for the system

$$x - 3y + z = 4$$
$$-x + 2y - 5z = 3$$
$$5x - 13y + 13z = 8$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ -1 & 2 & -5 & 3 \\ 5 & -13 & 13 & 8 \end{array}\right]$$

Write the system of equations from the augmented matrix

$$\begin{pmatrix} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 5 & -13 & 13 & 8 \end{pmatrix}$$

$x - 3y + z = 4$

$-y - 4z = 7$

$5x - 13y + 13z = 8$

Now - to manipulate our matrix:

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we use row operations

- interchange any 2 rows
- multiply all elements of a row by a nonzero real number
- add a multiple of one row to any other row

Our goal - is row echelon form (REF)

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

if there are any rows with all 0's they are at the bottom

or better yet - reduced row echelon form (RREF) $\rightarrow$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Row operations

$$x-2y+z=7$$

$$3x-5y+z=14$$

$$2x-2y-z=3$$

Notation:

1. R_{ij} means exchange rows i and j
2. kR_i means multiply ith row by k
3. $kR_i + R_j$ means adding k times the ith row to the jth row

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 2 & -2 & -1 & 3 \end{array}\right] \xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 2 & -3 & -11 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{2R_3+R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{-R_3+R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

REF

$$\xrightarrow{2R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$(2,-1,3)$$

What is the difference between REF and RREF?

Solve using RREF:

$$x - 3y + z = 4$$

$$-y - 4z = 7$$

$$5x - 13y + 13z = 8$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 5 & -13 & 13 & 8 \end{array}\right) \xrightarrow{-5R_1+R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & -8 & 14 \\ 0 & 2 & 8 & -12 \end{array}\right] \xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

$-5 \quad 15 \quad -5 \quad -20$

No Solution

reduced row echelon form

what does this mean?

Solve using RREF:

$$x + y + z = 3$$
$$2x + y + 4z = 8$$
$$x + 2y - z = 1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & 4 & 8 \\ 1 & 2 & -1 & 1 \end{array}\right] \xrightarrow[-R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & 2 & 2 \\ 0 & 1 & -2 & -2 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[-R_2]{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 3z = 5$

$x = 5 - 3z$

$y - 2z = -2$

$y = 2z - 2$

$(5-3z, 2z-2, z)$