

7.3 Matrices and Systems

Solve the system:

$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

That wasn't fun!!

$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

Instead lets use matrices
to record the variables
and use row operations
to solve

Systems with Matrices

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$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array} \right) \text{ or } \left(\begin{array}{cccc} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array} \right)$$

Write the augmented matrix for the system

$$x - 3y + z = 4$$

$$-x + 2y - 5z = 3$$

$$5x - 13y + 13z = 8$$

Write the system of equations from the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 5 & -13 & 13 & 8 \end{array} \right)$$

Now - to manipulate our matrix:

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Back

we use row operations

- interchange any 2 rows
- multiply all elements of a row by a nonzero real number
- add a multiple of one row to any other row

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Our goal - is row echelon form (REF)

$$\begin{pmatrix} 1 & -1 & 2 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

if there are any rows with all 0's they are at the bottom

or better yet - reduced row echelon form (RREF)

$$\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Row operations

$$x - 2y + z = 7$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

Notation:

1. R_{ij} means exchange rows i and j
2. kR_i means multiply i th row by k
3. $kR_i + R_j$ means adding k times the i th row to the j th row

What is the difference between REF and RREF?

Solve using RREF: $x - 3y + z = 4$
 $-y - 4z = 7$
 $5x - 13y + 13z = 8$

$$\begin{pmatrix} 1 & -3 & 1 & 4 \\ 0 & -1 & -4 & 7 \\ 5 & -13 & 13 & 8 \end{pmatrix}$$

reduced row echelon form

what does this mean?

Solve using RREF:

$$x + y + z = 3$$

$$2x + y + 4z = 8$$

$$x + 2y - z = 1$$