7.2 Matrices

#57

matrix - a rectangular array of variables and/or constants in horizontal rows and vertical columns enclosed by brackets

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

dimensions (order): m x n

#57- Back

m - number of rows

n - number of columns

each entry is called an element and uses double subscript to refer to its position

always named w/ capital letter

b_{47}

$$A_{3x3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix Vocabulary

#58

square matrix - same number of rows and columns

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

zero matrix - all elements are 0

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

equal matrices - same dimensions and corresponding elements are equal

Find the order of the Matricies

$$\begin{pmatrix} 1 & 7 \\ 4 & 3 \\ 8 & -7 \end{pmatrix}$$

3x2

$$\begin{pmatrix} 2 & -9 & 32 \\ -8 & 12 & 0 \end{pmatrix}$$

2x3

Scalar Multiplication #59

scalar: real number

scalar multiplication:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$5\begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix} = \begin{bmatrix} 10 & 15 \\ -20 & 5 \end{bmatrix}$$

$2A$

$$2\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Addition and Subtraction:

#60

you can only add and subtract matrices of the **<u>same size</u>**, if they are of the same size you simply add elements in the same position with another element in the same position

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix} + \begin{pmatrix} 3 & -5 \\ -2 & 7 \end{pmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 5 & 4 \\ -3 & 6 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -5 \\ -2 & 7 \\ -3 & 4 \end{pmatrix} = \begin{bmatrix} 2 & 9 \\ -1 & -1 \\ 3 & -2 \end{bmatrix}$$

Matrix Multiplication: #61

can only be done when the columns of the first matches the number of rows of the second

not communative

$AB \neq BA$

$2 \times 3 \quad 3 \times 2$

these have to match

the result will be the size of the rows of the first by columns of the second

$7 \times 4 \cdot 4 \times 3$

$3 \times 4 \cdot 7 \times 4$

$3 \times 2 \quad 3 \times 2$

these 2 matrices can't be multiplied

Does the product of AB exist, if yes, what is the size of the product?

AB: 2x3 · 3x2 ✓

2x2

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \end{pmatrix} \quad 2\text{x}3$$

BA: 3x2 · 2x3 ✓

3x3

$$B = \begin{pmatrix} 0 & 2 \\ -1 & 5 \\ -2 & 3 \end{pmatrix} \quad 3\text{x}2$$

Does the product of BA exist, if yes, what is the size of the product?

Matrix Multiplication Defined

#61-back

Matrix multiplication: is the product of each element in the row of the first with each element in the column of the second

$$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \bullet \underset{2\times 3}{\begin{bmatrix} e & f & g \\ \overset{h}{\cancel{g}} & \overset{i}{\cancel{h}} & j \end{bmatrix}}$$

rows by columns

$$= \begin{bmatrix} ae+bh & af+bi & ag+bj \\ ce+dh & cf+di & cg+dj \end{bmatrix}$$

order matters

$$\underset{2\times2}{\begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}} \bullet \underset{2\times2}{\begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}}$$

$$\begin{bmatrix} 15+6 & 3+4 \\ -5-6 & -1-4 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 7 \\ -11 & -5 \end{bmatrix}$$

$$\underset{2\times3}{\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \end{pmatrix}} \cdot \underset{3\times2}{\begin{pmatrix} 0 & 2 \\ -1 & 5 \\ -2 & 3 \end{pmatrix}}$$

$$\begin{bmatrix} 0+2-6 & 2-10+9 \\ 0+3-8 & 4-15+12 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 1 \\ -5 & 1 \end{bmatrix}$$

Calculator:

$$\begin{pmatrix} 1 & 3 & -1 \\ 4 & 2 & 7 \\ -2 & 0 & 5 \end{pmatrix} \bullet \begin{pmatrix} 10 & -3 & 1 \\ 4 & 3 & 5 \\ 2 & 0 & -2 \end{pmatrix}$$

Application Problems:sometimes when working an application problem, you need totranspose the matrix

transpose - switch the rows and columns

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 7 & 5 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 & -1 \\ 4 & 2 & 7 \\ -2 & 0 & 5 \end{pmatrix}$$

Identity Matrix: (I) is a square matrix with elements of 1 on the main diagonal and all other elements are 0.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$AI = A$

$3\left(\frac{1}{3}\right) = 1$

Inverse Matrix: a square matrix such that AB = BA = I, then B is the inverse of A.

$$B = A^{-1}$$

if a matrix does not have an inverse - it is a singular matrix

Verify that the matrices are inverses:

$$A = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

Inverse of a 2x2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \boxed{\frac{1}{\boxed{ad-bc}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

ad-bc is the determinant of a 2 x 2 matrix

determinants $\neq 0$ then the matrix has an inverse (non-singular)

Find the inverse of:

$$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$

$6 - 4 = \boxed{2}$

$$\frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}}$$