7.2 Matrices

matrix - a rectangular array of variables and/or constants in horizontal rows and vertical columns enclosed by brackets

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

dimensions (order): m x n

m - number of rows

n - number of columns

each entry is called an element and uses double subscript to refer to its position

always named w/ capital letter

$$A_{3x3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix Vocabulary

#58

square matrix - same number of rows and columns

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

zero matrix - all elements are 0

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

equal matrices - same dimensions and corresponding elements are equal

Find the order of the Matricies

$$\begin{pmatrix} 1 & 7 \\ 4 & 3 \\ 8 & -7 \end{pmatrix} \qquad \begin{pmatrix} 2 & -9 & 32 \\ -8 & 12 & 0 \end{pmatrix}$$

Scalar Multiplication

#59

scalar: real number

scalar multiplication:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$5\begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix} =$$

Addition and Subtraction:

#60

you can only add and subtract matrices of the **same size**, if they are of the same size you simply add elements in the same position with another element in the same position

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 5 & -6 \end{pmatrix} + \begin{pmatrix} 3 & -5 \\ -2 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 4 \\ -3 & 6 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -5 \\ -2 & 7 \\ -3 & 4 \end{pmatrix}$$

Matrix Multiplication: #61

can only be done when the columns of the first matches the number of rows of the second

$2 \times 3 \quad 3 \times 2$

these have to match

the result will be the size of the rows of the first by columns of the second

$3 \times 2 \quad 3 \times 2$

these 2 matrices can't be multiplied

Does the product of AB exist, if yes, what is the size of the product?

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 2 \\ -1 & 5 \\ -2 & 3 \end{pmatrix}$$

Does the product of BA exist, if yes, what is the size of the product?

Matrix Multiplication Defined

#61-back

Matrix multiplication: is the product of each element in the row of the first with each element in the column of the second

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \bullet \begin{bmatrix} e & f & g \\ g & h & j \end{bmatrix}$$

rows by columns

order matters

$$\begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix} \bullet \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ -1 & 5 \\ -2 & 3 \end{pmatrix}$$

Calculator:

$$\begin{pmatrix} 1 & 3 & -1 \\ 4 & 2 & 7 \\ -2 & 0 & 5 \end{pmatrix} \bullet \begin{pmatrix} 10 & -3 & 1 \\ 4 & 3 & 5 \\ 2 & 0 & -2 \end{pmatrix}$$

Application Problems: sometimes when working an application problem, you need to transpose the matrix

transpose - switch the rows and columns

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 7 & 5 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 & -1 \\ 4 & 2 & 7 \\ -2 & 0 & 5 \end{pmatrix}$$

Identity Matrix: (I) is a square matrix with elements of 1 on the main diagonal and all other elements are 0.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse Matrix: a square matrix such that AB = BA = I, then B is the inverse of A.

$$B = A^{-1}$$

if a matrix does not have an inverse - it is a singular matrix

Verify that the matrices are inverses:

$$A = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

Inverse of a 2x2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

ad-bc is the determinant of a 2 x 2 matrix

determinants $\neq 0$ then the matrix has an inverse (non-singular)

Find the inverse of:

$$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$