7.1 Solving Systems of Two Equations

\[
2x - y = 10 \\
3x + 2y = 1
\]

A Solution is an ordered pair \((x, y)\) that makes both equations true.

The two main methods for solving systems are substitution and elimination.
Solving by substitution:

Solving by substitution means we solve for one variable in one equation and then substitute that expression into the second equation to solve for the remaining variable.

\[
\begin{align*}
2x - y &= 10 \\
3x + 2y &= 1
\end{align*}
\]

\[
y = 2x - 10
\]

\[
3x + 2(2x - 10) = 1
\]

\[
3x + 4x - 20 = 1
\]

\[
7x = 21
\]

\[
x = 3
\]

\[
y = 2(3) - 10
\]

\[
y = 6 - 10
\]

\[
y = -4
\]

\[(3, -4)\]
Find the dimensions of a rectangular garden that has perimeter 100 ft and area of 300 ft.

\[
\begin{align*}
2x + 2y &= 100 \\
yx &= 300 \\
2(x+y) &= 100 \\
x+y &= 50 \\
x &= 50 - y \\
y(50-y) &= 300 \\
50y - y^2 &= 300 \\
0 &= y^2 - 50y + 300 \\
y &= \frac{50 \pm \sqrt{50^2 - 4(300)}}{2} \\
y &= 7, 43 \\
x &= 50 - y \\
x &= 7
\end{align*}
\]
Solving by Elimination

Solving by elimination means using basic operations we eliminate one of the variables when adding the equations together, then solve for the remaining variable.

\[ 3(2x + 3y = 5) \]
\[ 2(-3x + 5y = 21) \]
\[ + \]
\[ 6x + 9y = 15 \]
\[ -6x + 10y = 42 \]
\[ 0 + 19y = 57 \]
\[ y = 3 \]

\[ \frac{19y}{19} \]

\[ 2x + 3(3) = 5 \]
\[ 2x + 9 = 5 \]
\[ 2x = -4 \]
\[ x = -2 \]

\[ (-2, 3) \]
Solve:
\[
\begin{align*}
  x - 3y &= -2 \\
  2x - 6y &= 4
\end{align*}
\]
\[x = 3y - 2\]

\[
\begin{align*}
  2(3y-2) - 6y &= 4 \\
  6y - 4 - 6y &= 4 \\
  -4 &= 4
\end{align*}
\]

\boxed{\text{no solution}}
Solve:

\[ 3(4x - 5y = 2) \]

\[
\begin{align*}
-12x + 15y &= -6 \\
+ 12x - 15y &= 6 \\
\hline
0 + 0 &= 0
\end{align*}
\]

\[ 0 = 0 \]

\text{Infinite}
Possible Solutions:

One/two/three Solution(s): A unique ordered pair \((x, y)\) that satisfies both equations. Number of solutions depends on the degree: Linear- one, Quadratic - two, cubic - three

No Solution: There is no ordered pair \((x, y)\) that will make both equations true.

Infinitely many solutions: There are infinite ordered pairs \((x, y)\) that make both equations true.