6.5 Graphs of Polar Equations

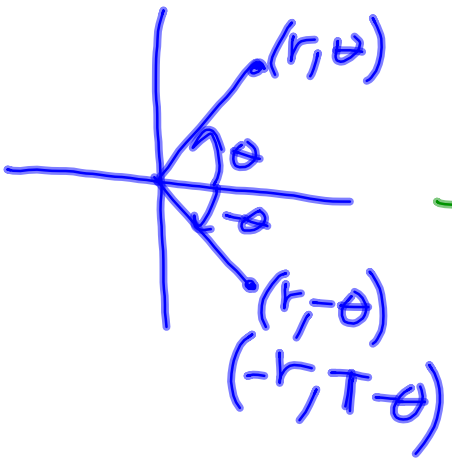
$$r = 5\cos 2\theta$$

$$r = 3 + 3\cos\theta$$

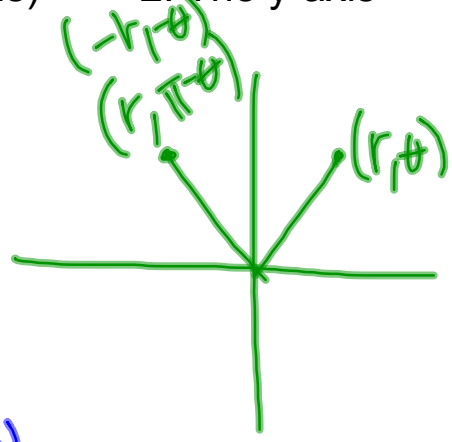
$$r = 5$$

Types of Symmetry

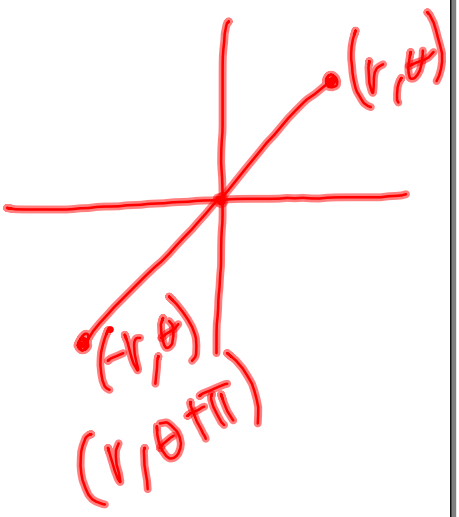
1. The x-axis (polar axis)



2. The y-axis



3. The origin (the pole)



Symmetry Test for Polar Equations

The graph of a polar equation has the indicated symmetry if when replaced you get an equivalent expression

Symmetry	Replace	By
1. The x-axis (polar axis)	(r,θ)	$(r,-\theta) or (-r,\pi-\theta)$
2. The y-axis	(r,θ)	$(-r,-\theta) or (r,\pi-\theta)$
3. The origin (the pole)	(r,θ)	$(-r,\theta) or (r,\theta+\pi)$

Find the Symmetry of $r = 4\sin 3\theta$

$\sin(-\theta) = -\sin\theta$

Y-axis

(r, θ) $(-r, -\theta)$

$-r = 4\sin(-3\theta)$

$-1(-r) = (-(4\sin 3\theta))-1$

Y-axis ✓

Symmetry $r = 4\sin 3\theta$

Rose Curves $r = a\sin n\theta$ $r = a\cos n\theta$

Analyze means find:

1. # of Petals

2. Length of Petals

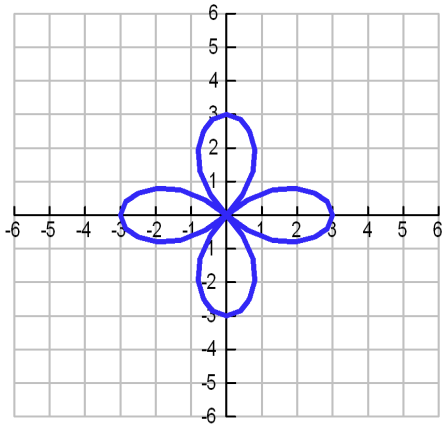
3. Domain

4. Range

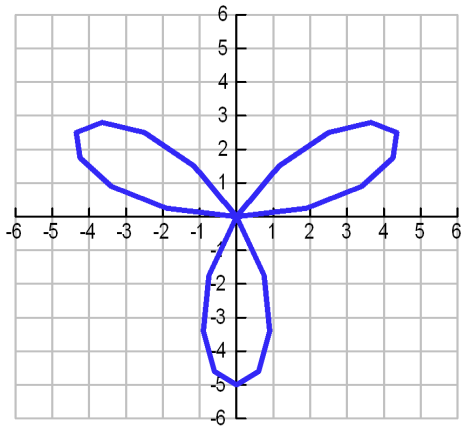
5. Symmetry

1. # of Petals

$r = 3\cos 2\theta$



$r = 5\sin 3\theta$



# of Petals:

2. Length of Petals

$r = a \sin n\theta$

Trace our Petals in the calculator

What do you find about the max and mins?

Length of Petals: $|a|$

3. Domain/ 4. Range

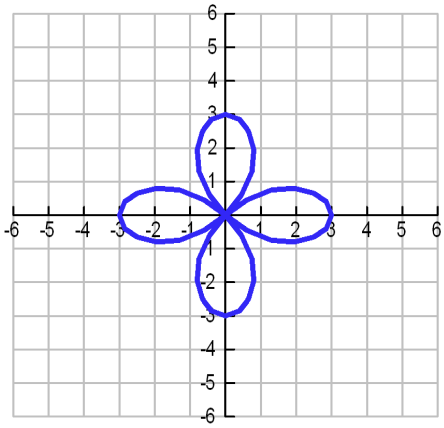
$r = a\sin(n\theta)$

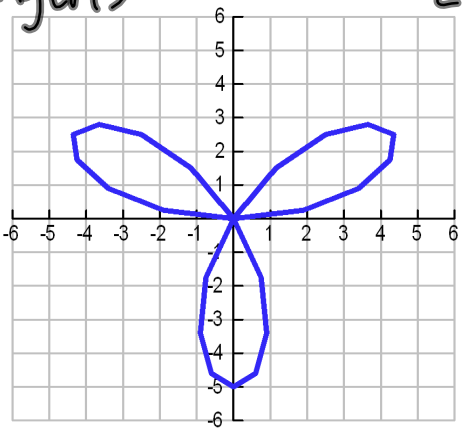
What do you put into a polar equation? (Domain)

angles all real #'s

What do you get out of a polar equation? (Range)

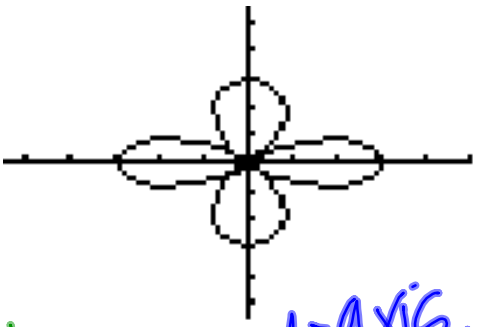
r lengths $[-|a|, |a|]$





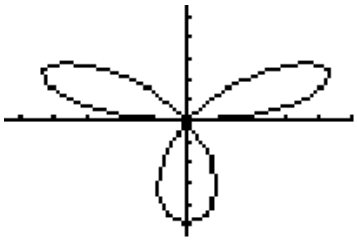
5. Symmetry

$r = 3\cos 2\theta$



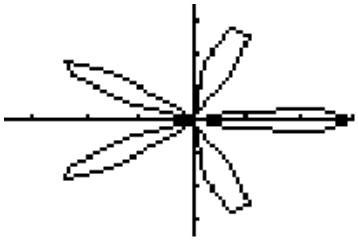
x-axis, y-axis, origin

$r = 3\sin 3\theta$



y-axis

$r = 3\cos 5\theta$



x-axis

What do you conclude about their symmetry?

Rose Curves

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$r = a\sin(n\theta)$ $r = a\cos(n\theta)$

# of Petals: n, if n is odd
2n, if n is even

Length of Petals: |a|

Domain: All real Numbers

Range: [-|a|, |a|]

Symmetry: n even, Symmetric about x, y -axis, and origin
n odd, $r = a\cos n\theta$ symmetric about x-axis
n odd, $r = a\sin n\theta$ symmetric about y-axis

Limaçon Curves

Lima son

pronounced: LEE-ma-sohn

$r = a \pm b\sin\theta$

$r = a \pm b\cos\theta$

Analyzing means find:

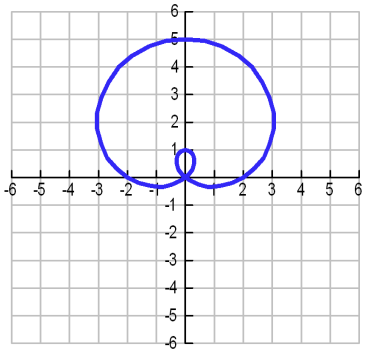
1. Type
2. Domain
3. Range
4. Symmetry

1. Type

$r = 2 + 3\sin\theta$

Looped Limaçon

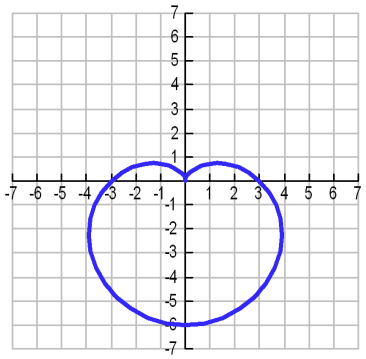
$\frac{a}{b} < 1$



$r = 3 - 3\sin\theta$

Cardiod

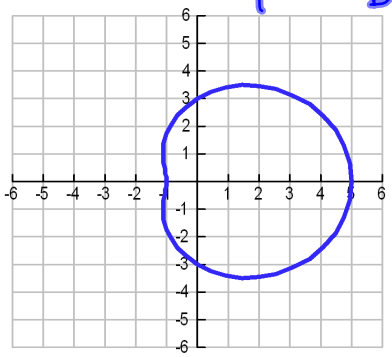
$\frac{a}{b} = 1$



$r = 3 + 2\cos\theta$

$1 < \frac{a}{b} < 2$

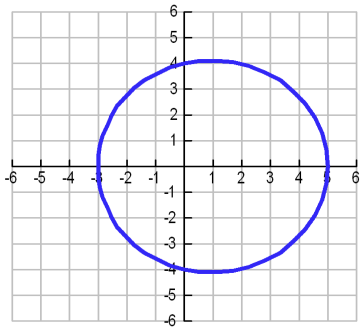
Dimpled Limaçon



$r = 4 + \cos\theta$

Convex Limaçon

$\frac{a}{b} \geq 2$



2/3 Domain and Range

$r = a \pm b\sin\theta$

What do you put into a polar equation? (Domain)

angles

What do you get out of a polar equation? (Range)

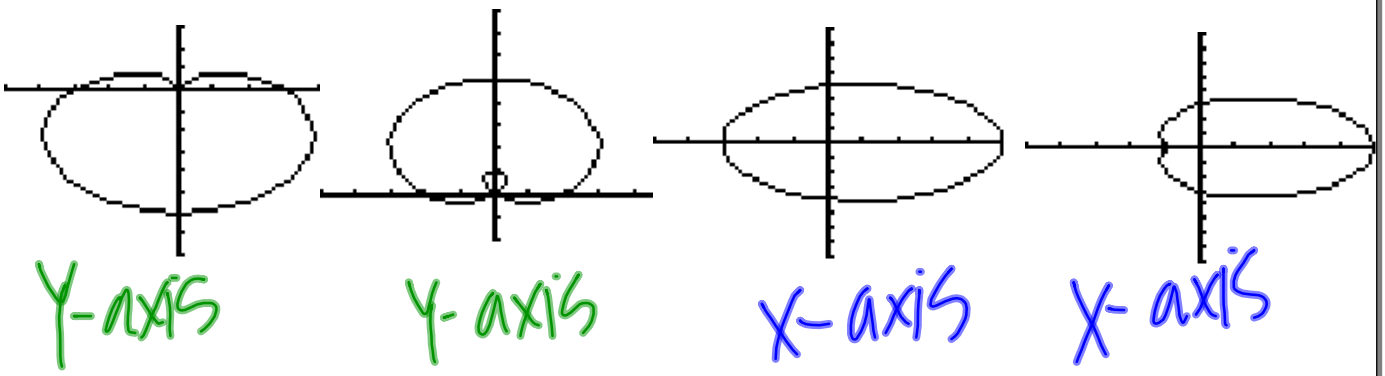
r-length

Domain: all real #'s

Range: $[a-b, a+b]$

4. Symmetry

$r = 3 - 3\sin\theta$ $r = 2 + 3\sin\theta$ $r = 4 + \cos\theta$ $r = 3 + 2\cos\theta$



Y-axis Y-axis X-axis X-axis

What do you conclude about their symmetry?

Limaçon Curves

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Domain: All real Numbers

Range: [a-b, a+b]

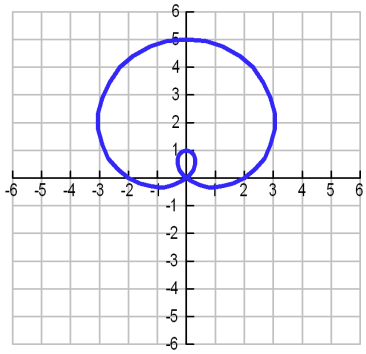
Symmetry: $r = a \pm b\cos\theta$ symmetric about x-axis
$r = a \pm b\sin\theta$ symmetric about y-axis

Type: $\frac{a}{b} < 1$: looped Limaçon
$\frac{a}{b} = 1$: Cardiod
$1 < \frac{a}{b} < 2$: Dimpled Limaçon
$\frac{a}{b} \geq 2$: convex Limaçon

1. Type

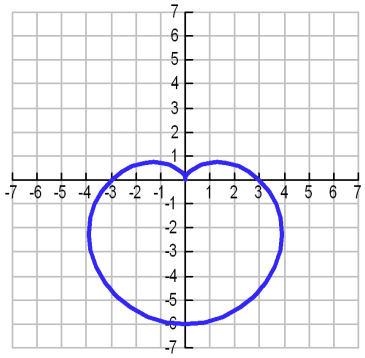
$r = 2 + 3\sin\theta$

Looped Limaçon



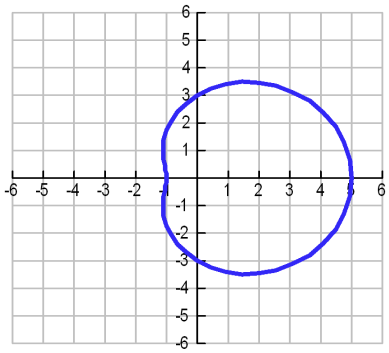
$r = 3 - 3\sin\theta$

Cardiod



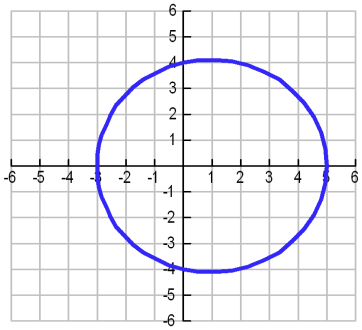
$r = 3 + 2\cos\theta$

Dimpled Limaçon



$r = 4 + \cos\theta$

Convex Limaçon



Archimedes Curve

$$r = \theta$$