6.4 Polar Coordinates

Polar Coordinate System: points are described by distance and direction

Distance is measured from a fixed point called the pole.

Direction is relative to a fixed ray with endpoint at the pole - called the polar axis.

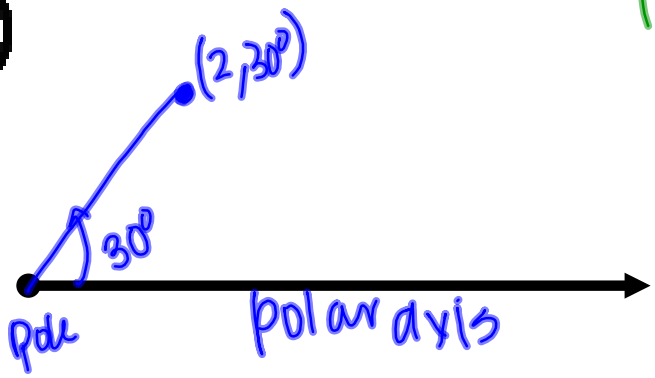
Polar Points:

(r, θ)

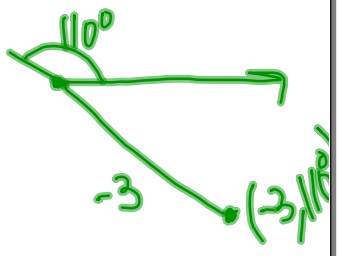
directed distance from the pole

directed angle measured from the polar axis

$(2, 30°)$



$(-3, 110°)$



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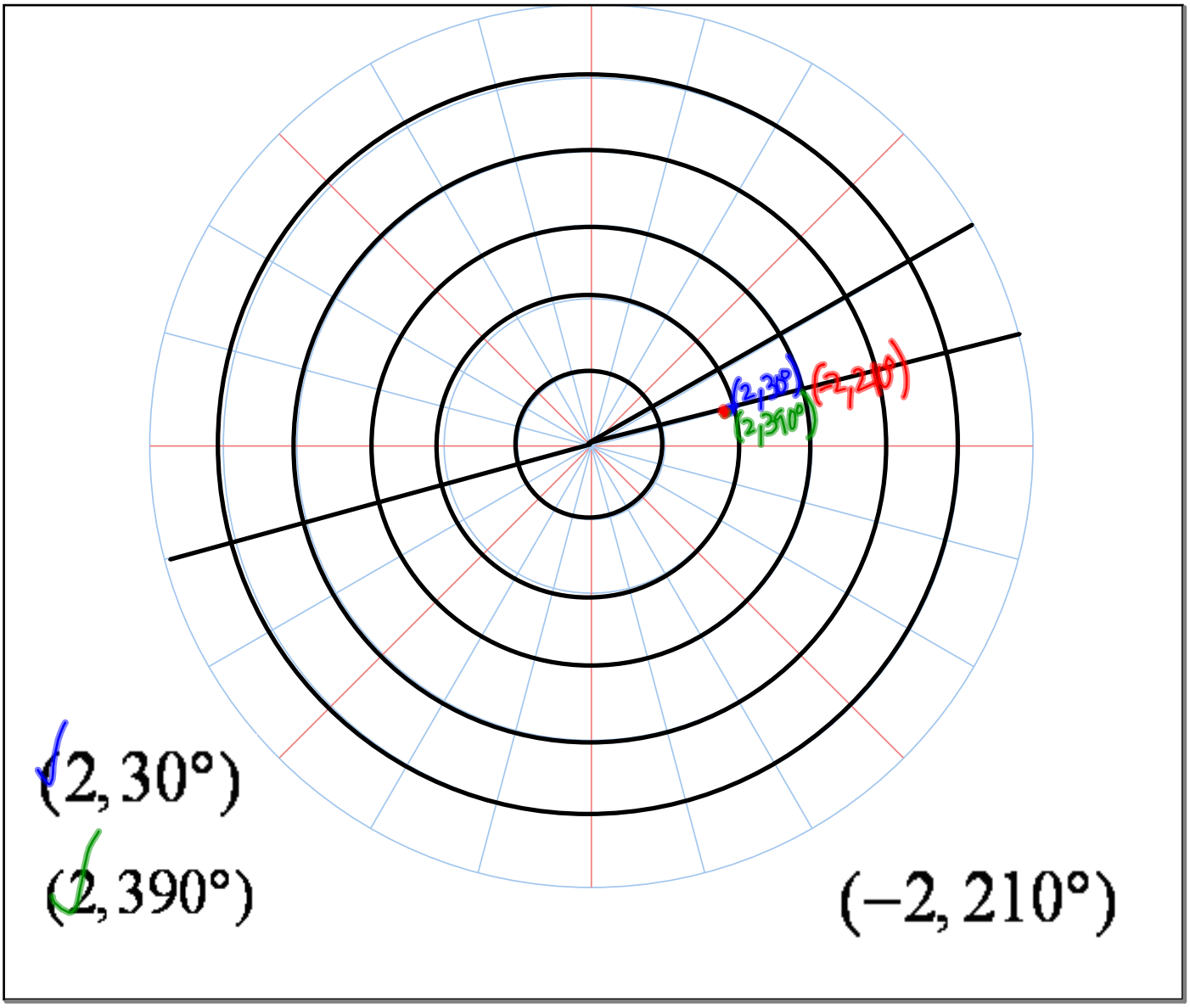


Each point can be expressed in infinitely many ways.

Using a negative radius, move backwards.

$(3, \pi/4)$

To express in general using radians: $(-3, \pi/4 + (2(0)+1)\pi)$

$(-3, \pi/4 + \pi)$

$(-3, 5\pi/4)$

$$(r, \theta + 2n\pi)$$

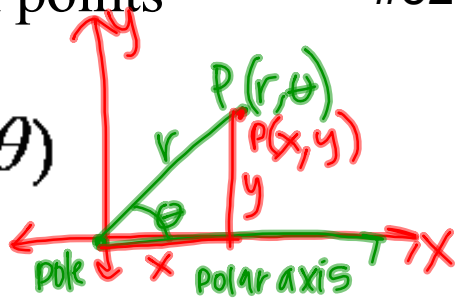
$$(-r, \theta + (2n+1)\pi)$$

Conversions with points

#52

Rectangular to Polar

$$(x, y) \rightarrow (r, \theta)$$



$$x^2 + y^2 = r^2$$ solve for r

Do we know any equations relating x, y, and r??

Do we know any equations relating x and y that will help us find an angle??

$$\tan\theta = \frac{y}{x}$$

solve for θ, refer to the ordered pair to get the correct quadrant

Give the polar coordinates for:

(0, 1)

$r^2 = x^2 + y^2$

$= 0^2 + 1^2$

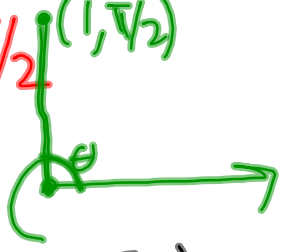
$r^2 = 1$

$r = \pm 1$

$\tan\theta = y/x$

$\tan\theta = 1/0$ undefined

$\pi/2, 3\pi/2$



$(1, \pi/2)\ (-1, 3\pi/2)$

(2, 2)

$r^2 = x^2 + y^2$

$= 2^2 + 2^2$

$r^2 = 8$

$r = \pm 2\sqrt{2}$

$\tan\theta = y/x$

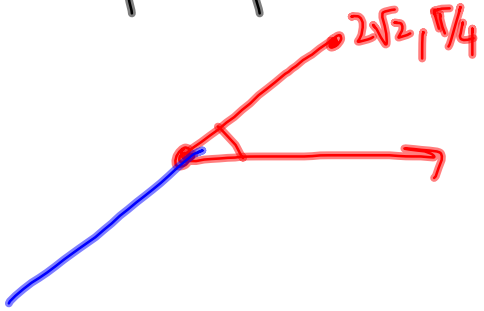
$= 2/2$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$(2\sqrt{2}, \pi/4)$

$(-2\sqrt{2}, \frac{5\pi}{4})$

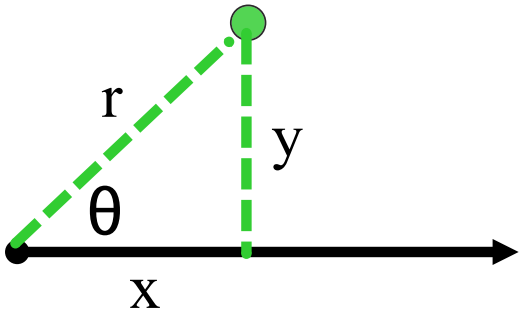


#52- back

Conversion: Polar to Rectangular

$$(r, \theta) \rightarrow (x, y)$$

this is the same idea as finding x and y components for initial velocity - just a different coordinate system



$$x = r\cos\theta$$

$$y = r\sin\theta$$

Give the rectangular coordinates for:

$\left(3, \frac{\pi}{2}\right)$

$x = r\cos\theta$

$= 3\cos\frac{\pi}{2}$

$= 3(0)$

$x = 0$

$y = r\sin\theta$

$= 3\sin\frac{\pi}{2}$

$= 3(1)$

$y = 3$

$(0, 3)$

$(1, \sqrt{3})$

$(2, 60°)$

$2\cos 60$

$2\left(\frac{1}{2}\right) = 1$

$2\sin 60$

$2\left(\frac{\sqrt{3}}{2}\right)$

$\sqrt{3}$

Conversions for Polar Equations

#53

r=

to convert equations use:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$$

and other identities as needed $\frac{1}{\sec\theta} = \cos\theta$

$$\frac{r}{\sec\theta} = \frac{5\sec\theta}{\sec\theta}$$

$$\frac{r}{\sec\theta} = 5$$

$$r\cos\theta = 5$$

$$\boxed{x = 5}$$

$3x + 4y = 5$

$x = r\cos\theta$ $y = r\sin\theta$

$3(r\cos\theta) + 4(r\sin\theta) = 5$

$r(3\cos\theta + 4\sin\theta) = 5$

$\frac{r(3\cos\theta + 4\sin\theta)}{3\cos\theta + 4\sin\theta} = \frac{5}{3\cos\theta + 4\sin\theta}$

$r = \frac{5}{3\cos\theta + 4\sin\theta}$

Graphing Polar in your calculator: