6.3 Parametric Equations

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equations that describe motion in terms of time because they have an additional variable (the parameter) - where an item is at a particular moment in time

there are 2 parts to a parametric equation - in terms of time

$x(t) =$ this is one set of equations

$y(t) =$

Vocabulary:

$x(t) = f(t)$ and $y(t)=g(t)$ are functions defined on an interval of t-values called a parametric curve

$x(t)$ and $y(t)$ are parametric equations

the variable t is the parameter

the interval of t-values is called parameter interval

When we give parametric equations and a parameter interval for the curve we have parametrized the curve.

Complete the table:

$0 \le t \le 4$

$x(t) = 3t$

$y(t) = 2t + 4$

t	0	1	2	3	4
x	0	3	6	9	12
y	4	6	8	10	12

Graphing Parametric Equations

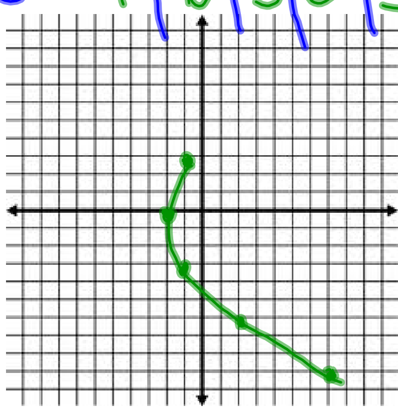
$$x = t^2 - 2$$

$$y = 3t$$

on the intervals:

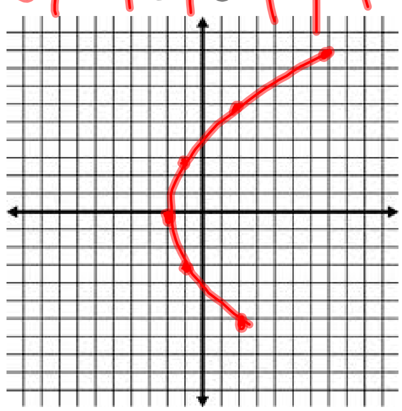
$-3 \le t \le 1$

t	-3	-2	-1	0	1
x	7	2	-1	-2	-1
y	-9	-6	-3	0	3



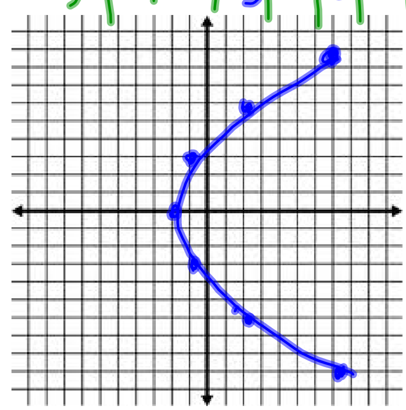
$-2 \le t \le 3$

t	-2	-1	0	1	2	3
x	2	-1	-2	-1	2	7
y	-6	-3	6	3	6	9



$-3 \le t \le 3$

t	-3	-2	-1	0	1	2	3
x	7	2	-1	-2	-1	2	7
y	-9	-6	-3	0	3	6	9



Graphing Parametric Equations

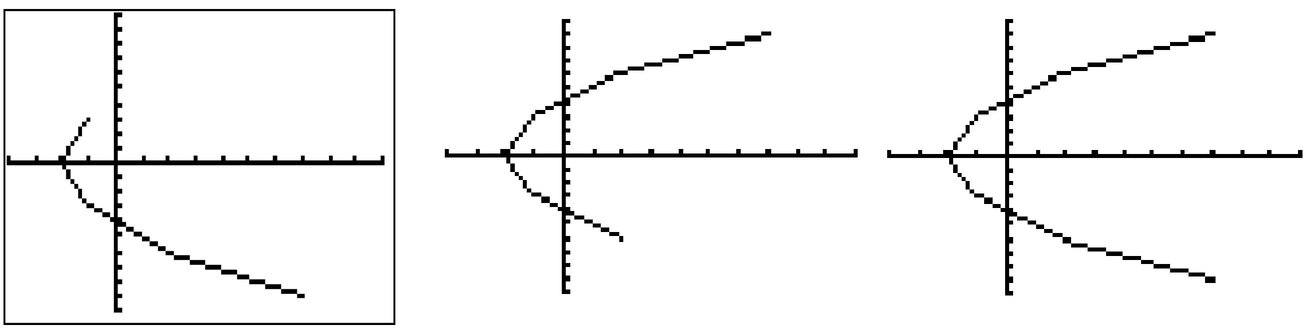
$$x = t^2 - 2$$

$$y = 3t$$

On your calculator:

on the intervals:

$-3 \leq t \leq 1$ $\quad$ $-2 \leq t \leq 3$ $\quad$ $-3 \leq t \leq 3$



Projectile Motion

A distress flare is shot straight up from a ship's bridge 75ft above the water with an initial velocity of 76ft/sec.

$[y(t)]\ h(t) = -16t^2 + v_0 t + h_0$

a) Use an equation to model the height of the flare as a function of time.

b) Use parametric mode to simulate the drop during the first 5 seconds.

c) After 4 seconds someone sees the flare. How high was the flare when it was seen? 123ft

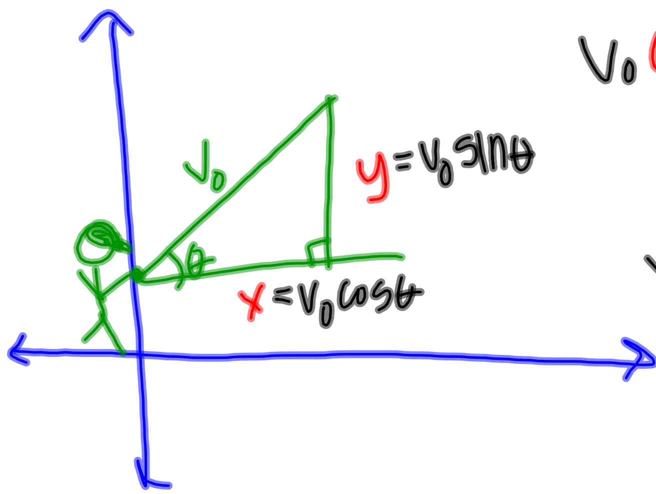
$x(t) = t \qquad y(t) = -16t^2 + 76t + 75$

$-16(4^2) + 76(4) + 75 = 123$

$x_2(t) = 5.5 \qquad y_2(t) = -16t^2 + 76t + 75$

But what about motion that is not straight up? But rather at an angle?

Taijuan Walker throws a baseball from a point $y_\circ$ feet above ground level with an initial speed of $v_\circ$ ft/sec at an angle θ with the horizontal. How do we represent the motion?



$$V_0 \cos\theta = \frac{x}{V_0} \cdot V_0$$

$$x = V_0 \cos\theta$$

$$V_0 \sin\theta = \frac{y}{V_0} \cdot V_0$$

$$y = V_0 \sin\theta$$

$$v = \langle V_0 \cos\theta, V_0 \sin\theta \rangle$$

$$h(t) = -16t^2 + v_o t + h_0$$

$$v = \langle v_0 \cos\theta, v_0 \sin\theta \rangle$$

$$\text{pathway} \begin{cases} x = (v_0 \cos\theta)t \\ y = -16t^2 + (v_0 \sin\theta)t + h_0 \end{cases}$$

Kevin hits a baseball at 3 ft above ground with an initial speed of 150 ft/sec. at an angle of 18°. Will the ball clear a 20 ft. fence that is 400 ft away? NO

$x_1(t) = (150\cos 18°)t$

$y_1(t) = -16t^2 + (150\sin 18)t + 3$

$t = 3$

$x_2(t) = 400$

$y_2(t) = 20(t/3)$

Motion

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$$h(t) = -16t^2 + v_o t + h_0$$

Used when objects are released at a 90 degree angle- Straight up

V_0 = initial velocity
h_0 = initial height

pathway $\begin{cases} x = (v_0 \cos\theta)t \\ y = -16t^2 + (v_0 \sin\theta)t + h_0 \end{cases}$

Used when objects are released at an angle θ with the horizontal

Parametric vs. Rectangular

Eliminate the parameter in the following equation:

$$x(t) = 3t$$
$$y(t) = 2t + 4$$

$x = 3t$ $\quad t = \frac{x}{3}$

$y = 2\left(\frac{x}{3}\right) + 4$

$y = \frac{2}{3}x + 4$

Eliminate the parameter in the following equation:

$$x = t^2 + 3$$

$$y = 4t$$

$x = \left(\frac{y}{4}\right)^2 + 3$

$t = \frac{y}{4}$

$x = t^2 + 3$

$-3 \qquad -3$

$x - 3 = t^2$

$\pm\sqrt{x-3} = t$

$y = 4\left(\pm\sqrt{x-3}\right)$

$y = 4\sqrt{x-3}$

$y = -4\sqrt{x-3}$