

6.3 Parametric Equations

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equations that describe motion in terms of time because they have an additional variable (the parameter) - where an item is at a particular moment in time

there are 2 parts to a parametric equation - in terms of time

$x(t) =$ **this is one set of equations**

$y(t) =$

Vocabulary:

$x(t) = f(t)$ and $y(t) = g(t)$ are functions defined on an interval of t -values called a parametric curve

$x(t)$ and $y(t)$ are parametric equations

the variable t is the parameter

the interval of t -values is called parameter interval

When we give parametric equations and a parameter interval for the curve we have parametrized the curve.

Complete the table:

$$x(t) = 3t$$

$$y(t) = 2t + 4$$

t	0	1	2	3	4

Graphing Parametric Equations

$$x = t^2 - 2$$

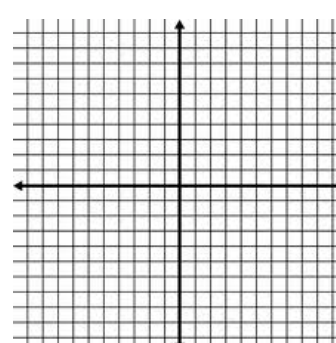
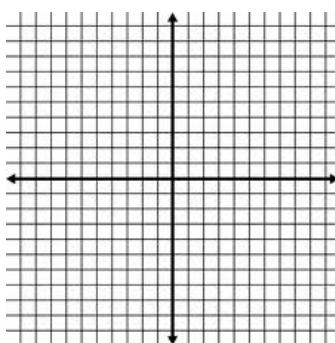
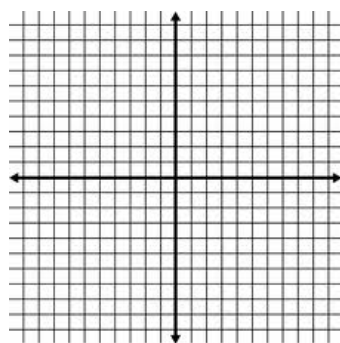
$$y = 3t$$

on the intervals:

$$-3 \leq t \leq 1$$

$$-2 \leq t \leq 3$$

$$-3 \leq t \leq 3$$



Graphing Parametric Equations

$$x = t^2 - 2$$

$$y = 3t$$

on the intervals:

$$-3 \leq t \leq 1$$

On your calculator:

$$-2 \leq t \leq 3$$

$$-3 \leq t \leq 3$$

Parametric vs. Rectangular

Eliminate the parameter in the following equation:

$$x(t) = 3t$$

$$y(t) = 2t + 4$$

Eliminate the parameter in the following equation:

$$x = t^2 + 3$$

$$y = 4t$$

Eliminate the parameter in the following equation:

$$x = 3 \cos t$$

$$y = 4 \sin t$$

to eliminate the parameter:

if only t is involved:

if t^2 is involved:

if \cos or \sin is involved:

Parametrization:

O - is the origin,
 P - is a point on the line
 AB

$A (-2, 3)$

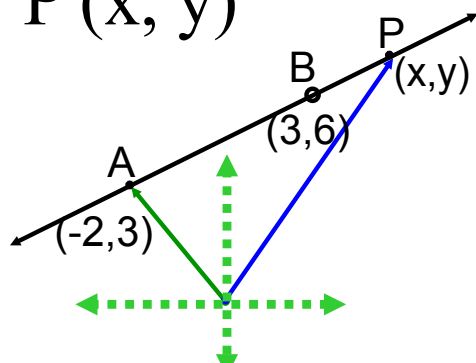
$B (3, 6)$

$P (x, y)$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + t \cdot \overrightarrow{AB}$$

$$\langle x, y \rangle = \langle -2, 3 \rangle + t \langle 3 - (-2), 6 - 3 \rangle$$



OR:

$$\mathbf{x} = \mathbf{x}_{tail} + t(\Delta\mathbf{x})$$

$$\mathbf{y} = \mathbf{y}_{tail} + t(\Delta\mathbf{y})$$

\overline{PQ}

P (-2, 3)

Q (3, 6)

P (3, 4)

Q (6, -3)

Projectile Motion

A distress flare is shot straight up from a ship's bridge 75ft above the water with an initial velocity of 76ft/sec.

- a) Use an equation to model the height of the flare as a function of time.
- b) Use parametric mode to simulate the drop during the first 5 seconds.
- c) After 4 seconds someone sees the flare. How high was the flare when it was seen?

But what about motion that is not straight up? But rather at an angle?

Taijuan Walker throws a baseball from a point y_0 feet above ground level with an initial speed of v_0 ft/sec at an angle θ with the horizontal. How do we represent the motion?

$$h(t) = -16t^2 + v_o t + h_0 \qquad v = \langle v_0 \cos \theta, v_0 \sin \theta \rangle$$

$$\text{pathway} \begin{cases} x = (v_0 \cos \theta)t \\ y = -16t^2 + (v_0 \sin \theta)t + h_0 \end{cases}$$

Kevin hits a baseball at 3 ft above ground with an initial speed of 150 ft/sec. at an angle of 18° . Will the ball clear a 20 ft. fence that is 400 ft away?

Motion

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$$h(t) = -16t^2 + v_o t + h_0$$

Used when objects are released at a 90 degree angle- Straight up

$$\text{pathway} \begin{cases} x = (v_0 \cos \theta)t \\ y = -16t^2 + (v_0 \sin \theta)t + h_0 \end{cases}$$

Used when objects are released at an angle θ with the horizontal