Find the distance from M to N, given M(-4, 7) and N(-1, 5).

\[ \sqrt{(-4+1)^2 + (7-5)^2} \]

\[ \sqrt{3^2 + 2^2} \]

\[ \sqrt{13} \]

6.1 Vectors

**scalar:** some quantities are represented by a single number - this indicates **magnitude** or size
distance, temperature, height

directed line segment: quantities with magnitude and **direction** such as force, velocity, acceleration (we use vectors)
\[(a, b)\] denotes a point in the coordinate plane.

Does \((a, b)\) have any direction? What about magnitude?

How can we make it have magnitude?

How can we give it direction?

Vectors are usually named with lower case bold letters

2 vectors are equal - if 2 directed line segments are equivalent (direction and magnitude are equal)
Vector Vocabulary

Standard Position: the arrow from the origin to the point \((a,b) = \langle a, b \rangle\)

Component form: shows the components of the vector \((\Delta x, \Delta y)\) from standard position (sometimes called the position vector)

Component form uses pointy parentheses
\[
\langle x_1, y_1 \rangle \text{ tail} \quad \langle x_2 - x_1, y_2 - y_1 \rangle
\]
\[
\langle x_2, y_2 \rangle \text{ head} \quad \langle \Delta x, \Delta y \rangle
\]

Magnitude: length of the vector (use distance formula)
\[
\| \mathbf{u} \| \quad \text{magnitude of vector } \mathbf{u}
\]

Zero vector: has 0 length and no direction

----

Find the component form and magnitude of:

given \(M(6, 5)\) and \(N(7, 12)\)

\[
\overrightarrow{MN} \quad \langle (7-6), (12-5) \rangle = \langle 1, 7 \rangle
\]
\[
\sqrt{1^2 + 7^2} = \sqrt{50}
\]

\[
\overrightarrow{NM} \quad \langle (6-7), (5-12) \rangle = \langle -1, -7 \rangle
\]
\[
\sqrt{(-1)^2 + (-7)^2} = \sqrt{50}
\]
vector addition and subtraction: add or subtract the components

scalars: distribute to both the x and y components

Example: \( \mathbf{u} = \langle 1, 4 \rangle \), \( \mathbf{v} = \langle -4, 5 \rangle \)

\[ 3\mathbf{v} = \langle -12, 15 \rangle \quad \mathbf{u} - \mathbf{v} = \langle 5, -1 \rangle \]

\[ \mathbf{u} + \mathbf{v} = \langle -3, 9 \rangle \quad 2\mathbf{u} - 3\mathbf{v} = \langle 14, -7 \rangle \]
Unit Vector: vector with magnitude of 1

\[ \frac{u}{|u|} \]

Horizontal \quad Vertical

**Standard Unit Vectors:**
\[ i = \langle 1, 0 \rangle \quad j = \langle 0, 1 \rangle \]

All vectors can be written using a linear combination of the standard unit vectors:
\[ v = \langle a, b \rangle \quad v = ai + bj \]
\[ v = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle \]
\[ v = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle \]
\[ v = \langle a, b \rangle \]

**Example:** \[ u = \langle -3, 2 \rangle \]

Find the unit vector and write it in linear combination form:
\[ |u| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13} \]

Unit vector:
\[ \left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle \]

\[ u = \frac{3}{\sqrt{13}}i + \frac{2}{\sqrt{13}}j \]
Applications of Vectors #233

Direction Angles: angles measured on unit circle in standard position (from the +x axis)

Given the direction angle and the magnitude, find the components of the vector:

\[ x = 15 \cos 30^\circ = 15 \cdot \frac{\sqrt{3}}{2}, \quad y = 15 \sin 30^\circ = \frac{15}{2} \]

\[ u = \langle u \cos \theta, u \sin \theta \rangle \]

Find the magnitude and direction of \( u \):

\[ u = \langle 3, 2 \rangle \]

\[ \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} \]

\[ \frac{3}{\sqrt{13}} = \cos \theta \left( \frac{3}{\sqrt{13}} \right) = 3 \theta 7^\circ \]
Find the magnitude and direction of $u$:

$$u = \langle -2, -5 \rangle$$

$$|u| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$$

$$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{29}}\right) = 248.2^\circ$$

**Bearings vs. Direction Angles**

You have to convert from bearings to direction angles 1st!!

An airplane is flying at a bearing of $170^\circ$ at 460 mph. Find the component form of the velocity of the plane.
The wind is blowing with the bearing of $200^\circ$ at 80 mph. Find the component form of the velocity of the wind.

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