5.4 Multiple Angle Identities

Double Angle

$\sin 2x = \underline{2\sin x\cos x}$

sin 60 = sin(2·30)

2 sin 30 cos 30

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

Power Reducing:

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$

$$\frac{\frac{1-\cos 2x}{2}}{\frac{1+\cos 2x}{2}}$$

$$\frac{1-\cos 2x}{\cancel{2}} \cdot \frac{\cancel{2}}{1+\cos 2x}$$

$$\frac{1-\cos 2x}{1+\cos 2x}$$

Half Angle

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\sin 60 = \sin\frac{120}{2}$$

$$\pm\sqrt{\frac{1-\cos 120}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{1-\cos x}{\sin x}$$

$$= \frac{\sin x}{1+\cos x}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

Use the half angle identities to find the exact value without a calculator

$$\sin 15^\circ = \sin\frac{30}{2} = \pm\sqrt{\frac{1-\cos 30}{2}} = \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}}$$

$$\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{1}{4}(2-\sqrt{3})}$$

$$\boxed{\frac{1}{2}\sqrt{2-\sqrt{3}}}$$

Prove: $(\sin x + \cos x)^2 = 1 + \sin 2x$

$(\sin x + \cos x)(\sin x + \cos x) = 1 + \sin 2x$

$\boxed{\sin^2 x} + \underline{\sin x \cos x + \cos x \sin x} + \boxed{\cos^2 x} = 1 + \sin 2x$

$1 + 2\sin x \cos x = 1 + \sin 2x$

$1 + \sin 2x = 1 + \sin 2x$

$1 + 2\sin x \cos x = 1 + 2\sin x \cos x$

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$$\frac{2\tan x}{\sec^2 x} = \sin 2x$$

$$\frac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{1}{\cos^2 x}} = \sin 2x$$

$$\frac{2\sin x}{\cancel{\cos x}} \cdot \frac{\cos^{\cancel{2}} x}{1} = \sin 2x$$

$2\sin x \cos x = \sin 2x$

$\sin 2x = \sin 2x$

Solve the equation:

$\sin 2x + \cos x = 0$

1. one side to be 0
2. Factor a common term

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad 2\sin x + 1 = 0$

$\qquad\qquad\qquad\quad -1 \;\; -1$

$x = \pi/2, 3\pi/2 \qquad 2\sin x = -1$

$\sin x = -1/2$

$x = \frac{11\pi}{6}, \frac{7\pi}{6}$

Use the power reducing formula to reduce:

$\cos^4 x = (\cos^2 x)(\cos^2 x)$

$= \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$= \frac{1 + \overbrace{1\cos 2x + \cos 2x}^{2\cos 2x} + \cos^2 2x}{4}$

$= \frac{1}{4} + \frac{2\cos 2x}{4} + \frac{1}{4}\cos^2 2x$

$= \frac{1}{4} + \frac{2}{4}\cos 2x + \frac{1}{4}\cos^2 2x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\cos^2 2x = \frac{1+\cos 4x}{2}$

$= \frac{1}{4} + \frac{2}{4}\cos 2x + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$

$= \frac{1}{4} + \frac{2}{4}\cos 2x + \frac{1}{4}\left(\frac{1}{2} + \frac{1\cos 4x}{2}\right)$

$= \frac{2}{8} + \frac{4}{8}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$

$= \frac{3}{8} + \frac{4}{8}\cos 2x + \frac{1}{8}\cos 4x$

$= \frac{1}{8}(3 + 4\cos 2x + \cos 4x)$

Use the power reducing formula to reduce: