5.3 Sum & Difference Identities

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

Find the exact value of:

$\cos 105^\circ = \cos(45+60) = \cos 45 \cos 60 - \sin 45 \sin 60$

$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\sin 15^\circ = \sin(45-30) = \sin 45 \cos 30 \cdot \cos 45 \sin 30$

$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$\frac{\pi}{6} = \frac{2\pi}{12}$

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

$= \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}(1)} = \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}} = \cancel{\frac{3+\sqrt{3}}{3}} \cdot \cancel{\frac{3}{3-\sqrt{3}}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$\frac{(3+\sqrt{3})^2}{9-3} = \frac{(3+\sqrt{3})^2}{6}$

Write as the sin, cos, or tan of an angle:

$$\sin 50^\circ \cos 26^\circ - \cos 50^\circ \sin 26^\circ$$

$$\sin(50-26) = \sin 24^\circ$$

$$\cos 50^\circ \cos 26^\circ - \sin 50^\circ \sin 26^\circ$$

$$\cos(50+26) = \cos 76^\circ$$

$$\frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$\tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \tan\frac{\pi}{12}$$

Prove the identity:

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

1- use identity
2- Evaluate
3- Simplify

$= \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2}$

$= \cos x(0) + \sin x(1)$

$= \sin x$ ■

If one of the angles in a sum or difference is a quadrantal angle (multiple of 90 or $\pi/2$) then the sum-difference identities yield single-termed expressions.

We call these reduction formulas

$$\sin(x-y)+\sin(x+y)=2\sin x\cos y$$

$\sin x\cos y - \cancel{\cos x \sin y}$ +

$\sin x \cos y + \cancel{\cos x \sin y}$

$= 2\sin x\cos y$

Use your knowledge of identities to describe the transformation of these graphs.

$\cos(x-3)$

$\cos x \cos 3 + \sin x \sin 3$

$\cos(3-2x)$

$\cos(-2x+3)$

$\cos(-2(x-3/2))$

$\frac{2\pi}{2} = \pi$

Just like in 5.1 we can use identities to help us solve equations using trig

$$\cos 3x \cos x = \sin 3x \sin x$$

$$-\sin 3x \sin x \quad -\sin 3x \sin x$$

$$\cos 3x \cos x - \sin 3x \sin x = 0$$

$$\cos(3x + x) = 0$$

$$\cos(4x) = 0$$

$$\frac{4x}{4} = \frac{\frac{\pi}{2}}{4}$$

$$\boxed{x = \frac{\pi}{8}}$$

$$\sin(3x) = 3\cos^2 x \sin x - \sin^3 x$$

$\sin(2x+x) = \sin 2x \cos x + \cos 2x \sin x = \sin(x+x)\cos x + \cos(x+x)\sin x$

$[\sin x \cos x + \cos x \sin x]\cos x + [\cos x \cos x - \sin x \sin x]\sin x$

$\sin x \cos^2 x + \cos^2 x \sin x + \cos^2 x \sin x - \sin^3 x$

$3\cos^2 x \sin x - \sin^3 x$

Brock Jones