prove the algebraic identity by starting with the LHS expression and supplying a sequence of equivalent expressions that ends with the RHS expression

$$\tan x + \cot x = \sec x \csc x$$

$$\tan x + \cot x = \frac{\sin x}{\sin x}\cdot\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}$$

$$= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x}$$

$$= \csc x \sec x \; \blacksquare$$

General Strategies

- Begin with the more complicated expression and work toward the less complicated expression
- If no other move suggests itself, convert the entire expression to one involving sines and cosines
- Combine fractions by combining them over a common denominator.

Prove each identity:

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2$$

$$\frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2$$

$$\left(\frac{1}{\cos x}\right)^2 = \left(\frac{1}{\cos x}\right)^2 \blacksquare$$

$$\frac{1+\cot^2 x}{\csc^2 x} = 1$$

$$\frac{\csc^2 x}{\csc^2 x} = 1$$

$$1 = 1 \quad \blacksquare$$

$$\frac{\sec x}{\cos x} - 1 = \frac{\sin^2 x}{\cos^2 x}$$

$\frac{1}{\cos x} \cdot \frac{1}{\cos x}$

$1 + \tan^2 x = \sec^2 x$

$-1 \qquad -1$

$\tan^2 x = \sec^2 x - 1$

$\frac{1/\cos x}{\cos x} - 1 = \tan^2 x$

$\frac{1}{\cos^2 x} - 1 = \tan^2 x$

$\sec^2 x - 1 = \tan^2 x$

$\tan^2 x = \tan^2 x$ ■

$$\frac{\cot^2 x}{1+\csc x} = (\cot x)(\sec x - \tan x)$$

$x^2 - 49$

$(x-7)(x+7)$

$$\frac{\csc^2 x - 1}{1+\csc x} = \cot x \sec x - \tan x \cot x$$

$$\frac{\cancel{(1+\csc x)}(\csc x - 1)}{\cancel{1+\csc x}} = \left(\frac{\cancel{\cos x}}{\sin x}\right)\left(\frac{1}{\cancel{\cos x}}\right) - \cancel{\tan x}\left(\frac{1}{\cancel{\tan x}}\right)$$

$\csc x - 1 = \frac{1}{\sin x} - 1$

$\csc x - 1 = \csc x - 1$