5.1 Fundamental Identities

What does the word identity mean to you?

Identity:

#45

equality that is true for all values of the domain for both expressions as long as they are both defined

$$\tan\theta \bullet \cos\theta = \sin\theta$$

this is true for all θ, as long as $\sin\theta$, $\cos\theta$, and $\tan\theta$ are defined

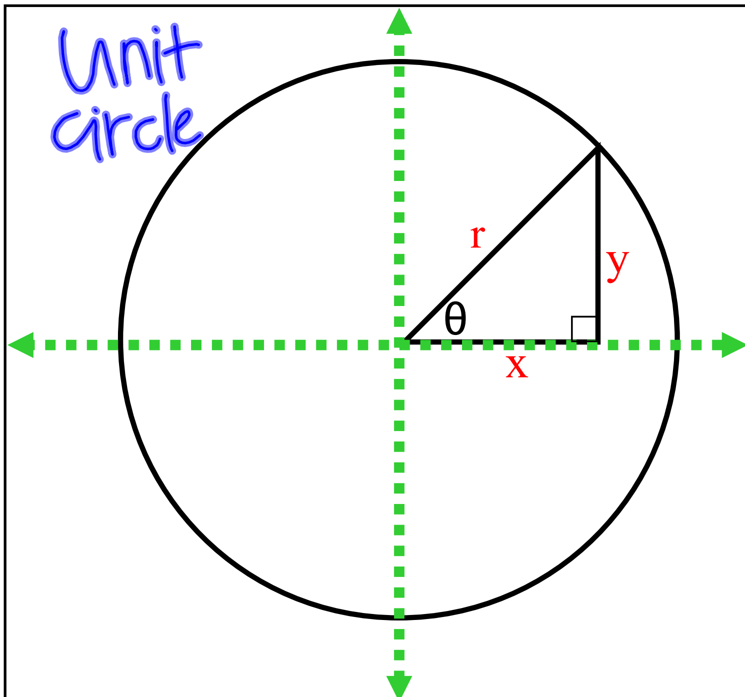
Reciprocal & Quotient Relationships

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Unit circle



$\sin\theta = y \quad \cos\theta = x \quad r = 1$

$x^2 + y^2 = r^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

$\cos^2\theta + \sin^2\theta = 1$

Pythagorean Relationships

$$x^2 + y^2 = r^2$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Pythagorean Relationships

$$\boxed{1+\tan^2\theta=\sec^2\theta}$$

$$1=\sec^2\theta-\tan^2\theta$$

$$\tan^2\theta=\sec^2\theta-1$$

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\boxed{1+\cot^2\theta=\csc^2\theta}$$

$$1=\csc^2\theta-\cot^2\theta$$

$$\cot^2\theta=\csc^2\theta-1$$

Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

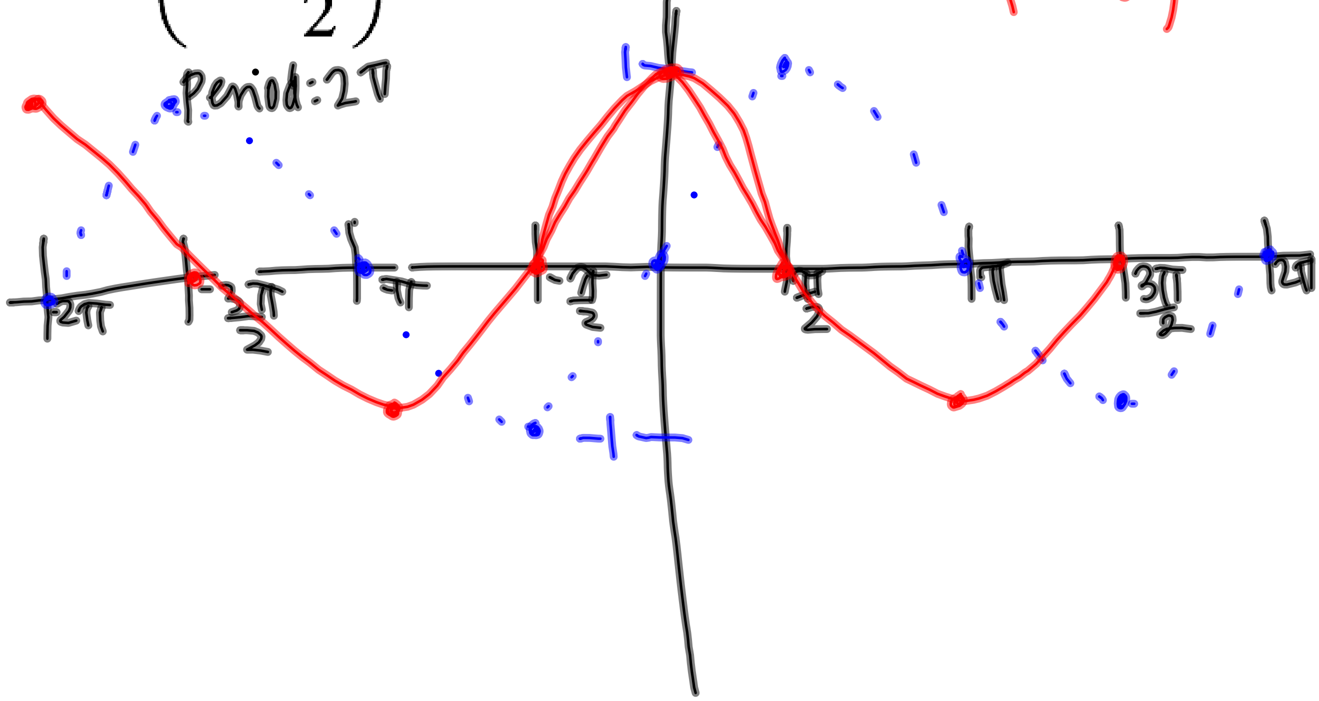
$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$y = \sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\cos x\left(x - \frac{\pi}{2}\right) = \sin x$

Period: 2π



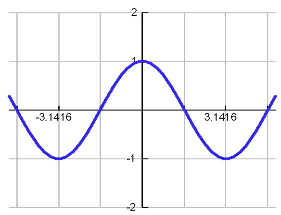
Co-Function Identities

$$\sin(\frac{\pi}{2}-\theta)=\cos\theta \qquad \cos(\frac{\pi}{2}-\theta)=\sin\theta$$

$$\tan(\frac{\pi}{2}-\theta)=\cot\theta \qquad \cot(\frac{\pi}{2}-\theta)=\tan\theta$$

$$\sec(\frac{\pi}{2}-\theta)=\csc\theta \qquad \csc(\frac{\pi}{2}-\theta)=\sec\theta$$

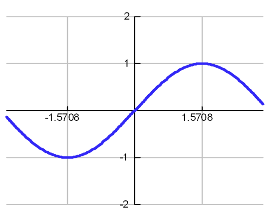
Odd/Even Identities



$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x$$

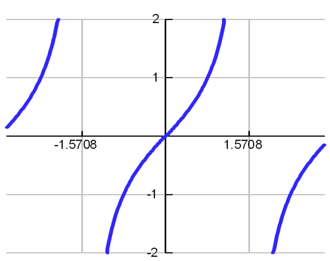
EVEN



$$\sin(-x) = -\sin x$$

$$\csc(-x) = -\csc x$$

ODD



$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

ODD

Simplify:

$$\sin x \csc(-x) = \sin x(-\csc x) = \cancel{\sin x}\left(-\frac{1}{\cancel{\sin x}}\right) = \boxed{-1}$$

$$\cot x \tan x = \left(\frac{1}{\cancel{\tan x}}\right)\cancel{\tan x} = \boxed{1}$$

Perfect Squares:

$x^2 - 8x + 16 = (x-4)^2$

$x^2 + 14x + 49$

$(x+7)^2$

$\sin^2 x - 10\sin x + 25 = y^2 - 10y + 25$

$\sin x = y$

$(y-5)^2 = (\sin x - 5)^2$

$\cos^2 x + 16\cos x + 64$

$(\cos x + 8)^2$

$\cos\theta = \theta$

Difference of Squares:

$x^2 - 16$ $(x+4)(x-4)$ $x^2 - 49$ $(x+7)(x-7)$

$1 - x^2$
$(1-x)(1+x)$

$1 - \sin^2 x = (1-\sin x)(1+\sin x)$

$\sin^2 x - \cos^2 x$ $(\sin x - \cos x)(\sin x + \cos x)$

Simplify:

$$\frac{1-\cos^2 x}{1+\cos x} = \frac{(1-\cos x)(1+\cancel{\cos x})}{1+\cancel{\cos x}} = \boxed{1-\cos x}$$

$$\frac{(1+\sin x)\,1}{(1-\sin x)(1+\sin x)} + \frac{1\,(1-\sin x)}{(1+\sin x)(1-\sin x)} = \frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x}$$

$$\frac{1+\cancel{\sin x}+1-\cancel{\sin x}}{1-\sin^2 x} = \frac{2}{1-\sin^2 x}$$

$$\frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = 2\left(\frac{1}{\cos^2 x}\right) = \boxed{2\sec^2 x}$$

Solve the Equation for $[0, 2\pi)$

$xy - y = x$

$y(x - 1) = x$

$\tan x \sin^2 x = \tan x$

$-\tan x \quad -\tan x$

$\tan x \sin^2 x - \tan x = 0$

$\tan x(\sin^2 x - 1) = 0$

$\downarrow \quad \downarrow$

$0 \quad 0$

$\tan x = 0$ $\boxed{x = 0, \pi}$

$\sin^2 x - 1 = 0$

$+1 \quad +1$

$\sin^2 x = 1$

$\sin x = \pm 1$ $\cancel{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

tan x is undefined @ $\frac{\pi}{2}$ & $\frac{3\pi}{2}$

How you write all solutions

Real #'s = $\mathbb{R}$

Complex #'s = $\mathbb{C}$

Integer = $\mathbb{Z}$ ←

Irrational #'s = $\mathbb{Q}$

$\pm\frac{\pi}{3} + 2n\pi,\ n \in \mathbb{Z}$

$n \mid n = 0, \pm 1 \ldots$