$3.6$ The Mathematics of Finance

Time = Money

When you borrow money you must pay interest.

When you put money into a savings account or loan money you can earn interest.

Interest is calculated using Exponential Functions!
Interest

Compounded Annually: \[ A = P(1 + r)^n \] (time years)

- Principle: Initial amount
- Amount after interest
- Rate (Decimal)
Suppose Blake invests $500 at 7% interest compounded annually. Find the value of his investment 10 years later.

\[ A = 500 \cdot \left(1 + \frac{0.07}{1}\right)^{10} \]

\[ A = 500 \cdot (1.7961) \]

\[ A = \$983.57 \]
Interest

Compounded Annually: \( A = P(1 + r)^n \)

Compounded \( k \) Times per year: \( A = P(1 + \frac{r}{k})^{kt} \)
Suppose Roberto invests $500 at 9% annual interest compounded monthly. Find the value of his investment 5 years later.

\[ A = 500 \left(1 + \frac{0.09}{12}\right)^{12 \cdot 5} \]

\[ A = 500(1.0075)^{60} \]

\[ \approx 7828.44 \]
Judy has $500 to invest at 9% annual interest compounded monthly. How long will it take her investment to grow to $3000?

\[
\frac{3000}{500} = \frac{3000}{500} = (1 + \frac{0.09}{12})^{12t}
\]

\[
t = \frac{\log(1.0075)}{\log(1.0075)} = 12t
\]

\[
\log 6 = \frac{\log(1.0075)}{12.372}
\]

\[
\approx 19.99 \text{ years}
\]
Stephen has $500 to invest. What annual interest rate compounded quarterly is required to double his money in 10 years?

\[
\frac{1000}{500} = 500 \left(1 + \frac{r}{4}\right)^{4 \times 10} \Rightarrow \frac{2}{500} = \left(1 + \frac{r}{4}\right)^4
\]

\[
\sqrt[4]{2} = 1 + \frac{r}{4}
\]

\[
4 \left(\sqrt[4]{2} - 1\right) = r
\]

\[
r = 0.0699
\]

\[
r = 6.99\%
\]
Interest

Compounded Annually: \[ A = P(1 + r)^n \]

Compounded \( k \) Times per year: \[ A = P \left(1 + \frac{r}{k}\right)^{kt} \]

Compounded Continuously: \[ A = Pe^{rt} \]
Suppose LaTasha has $100 to invest at 8% annual interest compounded continuously. Find the value of her investment after 1, 5, & 10 yrs.

\[ A = 100e^{0.08 \times 1} \]
\[ \approx 108.33 \]
\[ A = 100e^{0.08 \times 10} \]
\[ \approx 222.55 \]
\[ A = 100e^{0.08 \times 5} \]
\[ \approx 149.18 \]
Miss Mendel's saving account has a 0.05\% annual interest rate compounded continuously. If she has $2000 in her savings account now how long will it take her to make $500 in interest?

\[ A = Pe^{rt} \]

\[ \frac{2500}{2000} = e^{0.005t} \]

\[ \ln \frac{5}{4} = 0.005t \]

\[ t \approx 446.28 \]