$$ 3.6 The Mathematics of Finance $$

Time = Money

When you borrow money you must pay interest.

When you put money into a savings account or loan money you can earn interest.

Interest is calculated using Exponential Functions!

Interest

#39

Compounded Annually: $A = P(1+r)^n$

Principle : Initial amount

(time years)

Amount
After intrest

rate (Decimal)

Suppose Blake invests $500 at 7% interest compounded annually. Find the value of his investment 10 years later.

$$A = 500 (1 + .07)^{10}$$

$$A = 500 (1.97)$$

$$A = \$985$$

$$983.57$$

Interest

#39

Compounded Annually: $A = P(1+r)^n$

Compounded k Times per year: $A = P(1+\dfrac{r}{k})^{kt}$

Suppose Roberto invests \$500 at 9% annual interest compounded monthly. Find the value of his investment 5 years later.

$$A = 500\left(1 + \frac{.09}{12}\right)^{12 \cdot 5}$$

$$A = 500(1.0075)^{60}$$

~~6228.40~~ \$782.84

Judy has $500 to invest at 9% annual interest compounded monthly. How long will it take her investment to grow to $3000?

$$\frac{3000}{500} = \frac{500}{500}\left(1 + \frac{.09}{12}\right)^{12t}$$

$$6 = \left(1 + \frac{.09}{12}\right)^{12t}$$

$$6 = (1.0075)^{12t}$$

$$\log_{1.0075} 6 = 12t$$

$$\frac{\log 6}{12 \log 1.0075} = t \approx 19.98$$

6.372
6 years 5 months
.372(12) 4.46

20 years

Stephen has $500 to invest. What annual interest rate compounded quarterly is required to double his money in 10 years?

$$\frac{1000}{500} = \frac{500\left(1+\frac{r}{4}\right)^{4(10)}}{500}$$

$$\sqrt[40]{2} = \sqrt[40]{\left(1+\frac{r}{4}\right)^{40}}$$

$$\sqrt[40]{2} = 1+\frac{r}{4}$$

$$\sqrt[40]{2} - 1 = \frac{r}{4}$$

$$4\left(\sqrt[40]{2}-1\right) = r$$

$$r = .0699$$

$$r = 6.99\%$$

Interest

#39

Compounded Annually: $A = P(1+r)^n$

Compounded k Times per year: $A = P(1+\dfrac{r}{k})^{kt}$

Compounded Continuously: $A = Pe^{rt}$

Suppose LaTasha has $100 to invest at 8% annual interest compounded continuously. Find the value of her investment after 1, 5, & 10 yrs.

$$A = 100e^{.08 \times 1}$$
$108.33

$$A = 100e^{.08 \cdot 10}$$

$$A = 222.55$$

$$A = 100e^{.08 \times 5}$$
$149.18

Miss Mendel's saving account has a 0.05% annual interest rate compounded continuously. If she has $2000 in her savings account now how long will it take her to make $500 in interest?

$$A = Pe^{rt}$$

$$\frac{2500}{2000} = \frac{2000e^{.0005t}}{2000}$$

$$\ln 1.25 = \ln e^{.0005t}$$

$$\frac{\ln 1.25}{.0005} = \frac{.0005t}{.0005}$$

$$t \approx 446.287$$