3.5 Solving Exp & Log Equations

w/ matching bases

#38

If the bases are the same -

- Switch between Forms
- Properties of Logs & Exp.
- Graphing

☆ one-to-one property

IF $b^u = b^v$ then $u = v$

IF $\log_b u = \log_b v$ then $u = v$

GOAL: make the bases Equal & apply 1:1 property

$$2\left(\frac{1}{3}\right)^{\frac{x}{2}} = 54$$

$$\frac{2\left(\frac{1}{3}\right)^{\frac{x}{2}}}{2} = \frac{54}{2}$$

$$\left(\frac{1}{3}\right)^{\frac{x}{2}} = 27$$

$$\left(\frac{1}{3}\right)^{\frac{x}{2}} = \left(\frac{1}{3}\right)^{-3}$$

$$\frac{x}{2} = -3$$

$$x = -6$$

$$8\left(\frac{1}{2}\right)^{\frac{x}{4}} = 1$$

$$\frac{8\left(\frac{1}{2}\right)^{\frac{x}{4}}}{8} = \frac{1}{8}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{4}} = \frac{1}{8}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{4}} = \left(\frac{1}{2}\right)^{3}$$

$$\frac{x}{4} = 3$$

$$x = 12$$

$$\ln(x+12) = 3\ln 2$$

$$\ln(x+12) = \ln 8$$

$$x+12 = 8$$

$$-12 \quad -12$$

$$x = -4$$

$$\log(x-2)+\log(x+7)=3\log 4$$

$\log 4^3$

$\log(x-2)(x+7) = \log 64$

$x^2+5x-14 = 64$

$-64 \quad -64$

$x^2+5x-78=0$

$x \approx \pm 6.6787$

Solving Exp & Log Equations

w/o matching bases

#38-back

When the bases don't match:

you must ~~use~~ convert to the other form:

SWITCH FORMS

INVERSE OPERATIONS

$\log_b a = x \quad b^x = a$

$\log a = x \quad 10^x = a$

$\ln a = x \quad e^x = a$

$.94^x = 5.4$

$\log_{.94} .94^x = \log_{.94} 5.4$

$x = \log_{.94} 5.4$

$= \frac{\log 5.4}{\log .94}$

≈ -27.255

$8^x = 50$

$\log_8 8^x = \log_8 50$

$x = \frac{\log 50}{\log 8}$

$2^{3x} = 50$

$\log_2 2^{3x} = \log_2 50$

$3x = \log_2 50$

$3x = \frac{\log 50}{\log 2}$

≈ 1.881

$\log x^4 = 2$

$4\ln(x+7) - 5 = 1$

$+5 \quad +5$

$\frac{4\ln(x+7)}{4} = \frac{6}{4}$

$\ln(x+7) = \frac{3}{2}$

$x+7 = e^{(3/2)}$

$x = e^{(3/2)} - 7$

$$\log x^2 = 16$$

$\sqrt{x^2} = \sqrt{10^{16}} = (10^{16})^{1/2}$

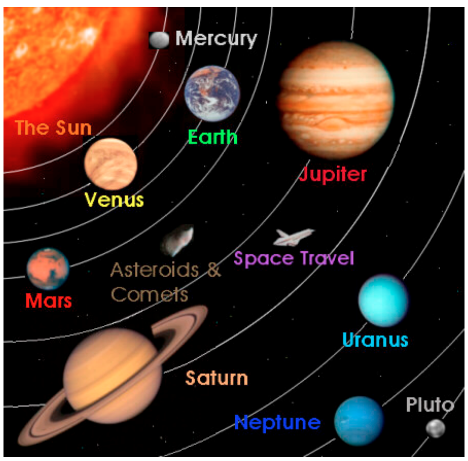
$x = \pm 10^8$

$2 \log x = 16$

$\log x = 8$

$x = 10^8$

Orders of Magnitude:



Mercury is 57.9 billion meters from the sun whereas Pluto is 5900 billion meters from the Sun.

a) Write the distance away from the sun for each planet in scientific notation.

M: 5.79×10^{10} P: 5.9×10^{12}

b) How many times farther is pluto away from the sun than Mercury?

100

c) take the common log of Mercury's distance and pluto's distance and compare the difference.

M: 10.76 P: 12.77

The common logarithm of a positive quantity is its order of magnitude

Allows us to compare sizes that have a wide range between them:

i.e.: Pluto's distance from the Sun is 2 orders of magnitude greater than Mercury's

A kilometer is 3 orders of magnitude longer than a meter

A dollar is 2 orders of magnitude greater than a penny

Logarithmic Scales are used in many important applications in your (yes, your) life.

Decibel Scale - Sound

Richter Scale - Earthquakes

Octave Scale - Music

F-Scale in Photography

pH Scale - Acidity

Brightness of Stars -

Ban and Deciban -

Palermo Technical Impact Hazard Scale

Decibel Scale

Source	Intensity	Intensity Level	# of Times Greater Than TOH
Threshold of Hearing (TOH)	$1*10^{-12}$ W/m^2	0 dB	10^0
Rustling Leaves	$1*10^{-11}$ W/m^2	10 dB	10^1
Whisper	$1*10^{-10}$ W/m^2	20 dB	10^2
Normal Conversation	$1*10^{-6}$ W/m^2	60 dB	10^6
Busy Street Traffic	$1*10^{-5}$ W/m^2	70 dB	10^7
Vacuum Cleaner	$1*10^{-4}$ W/m^2	80 dB	10^8
Large Orchestra	$6.3*10^{-3}$ W/m^2	98 dB	$10^{9.8}$
Walkman at Maximum Level	$1*10^{-2}$ W/m^2	100 dB	10^{10}
Front Rows of Rock Concert	$1*10^{-1}$ W/m^2	110 dB	10^{11}
Threshold of Pain	$1*10^{1}$ W/m^2	130 dB	10^{13}
Military Jet Takeoff	$1*10^{2}$ W/m^2	140 dB	10^{14}
Instant Perforation of Eardrum	$1*10^{4}$ W/m^2	160 dB	10^{16}

Richter Scale

Magnitude	Description	Earthquake effects	Frequency of occurrence
Less than 2.0	Micro	Micro earthquakes, not felt.[13]	Continual
2.0–2.9	Minor	Generally not felt, but recorded.	1,300,000 per year (est.)
3.0–3.9		Often felt, but rarely causes damage.	130,000 per year (est.)
4.0–4.9	Light	Noticeable shaking of indoor items, rattling noises. Significant damage unlikely.	13,000 per year (est.)
5.0–5.9	Moderate	Can cause major damage to poorly constructed buildings over small regions. At most slight damage to well-designed buildings.	1,319 per year
6.0–6.9	Strong	Can be destructive in areas up to about 160 kilometres (99 mi) across in populated areas.	134 per year
7.0–7.9	Major	Can cause serious damage over larger areas.	15 per year
8.0–8.9	Great	Can cause serious damage in areas several hundred kilometres across.	1 per year
9.0–9.9		Devastating in areas several thousand kilometres across.	1 per 10 years (est.)
10.0+	Massive	Never recorded, widespread devastation across very large areas; see below for equivalent seismic energy yield.	Extremely rare (Unknown/May not be possible)