2.8 Solving Inequalities in One Variable

when solving an inequality - your answer is the $x$ values for where the function ($y$ values) meets the given conditions

$$f(x) > 0$$

report the $x$ values for where the $y$'s are greater than zero

$$(-\infty, -1.5) \cup (1, \infty)$$
A full graph is not needed to do this so we use a sign chart.
(A sign chart shows only the items related to the signs of the function.)

What are the important items in a graph?

\[ x - \text{intercepts} \]
\[
(x + 3)(x - 2)(x + 2)^2 \geq 0
\]

\[ (-\infty, -3] \cup [2, \infty) \]

\[ (-)(-)(+) (+)(-)(+) (+)(-) (+)(+) (+) \]

\[ (+) -3 \] \[ (-) -2 \] \[ (-) \] \[ 2 \] \[ (+) \]

\[ \sqrt{-3}: m = 1 \]
\[ \text{Straight crosses: Sign changes} \]

\[ \sqrt{-2}: m = 2 \]
\[ \text{Tangent: doesn't: No sign change} \]

\[ \sqrt{2}: m = 1 \]
Polynomial Inequalities

goal: solving where the polynomial is (+) or (-)

Everything on 1 side and factored

Find all x-intercepts

Plot using open & closed holes according to the inequality sign

Find the signs of the graph in the intervals b/w the intercepts (use a value in the interval)

Answer: the intervals according to the inequality signs (use the union symbol if more than 1 interval)
\[-(x^2 + 3)(x - 5)^2 < 0\]

\[\begin{align*}
(-\infty, 5) \cup (5, \infty) \\
(-)(+)(+) \quad (+)(-)(+)
\end{align*}\]

\[\begin{align*}
(x - 3)(x + 4)^2(x - 1)^3 & \geq 0 \\
(3, -4) \cup [3, 00) \\
(-\infty, -4) \cup (4, 1) \cup (3, 00)
\end{align*}\]

Degree: 6 → Even

LC: +
Solve the polynomial inequality graphically.

\[ 2x^3 - 3x^2 - 5x + 5 < 0 \]

When asked to solve graphically - use your calculator to find the x-intercepts - then give the appropriate intervals based on the question.

\((-\infty, -1.44) \cup (0.82, 2.12)\)

\[2^{nd} \text{ calc: zero}\]
Making Sign Chart for a Rational Function

\[
\frac{(2x + 1)}{(x + 3)(x - 1)} \rightarrow \text{X-Int.}
\]

\[
(x + 3)(x - 1) \rightarrow \text{V.A}
\]

Include X-Int \& V.A.
Rational Inequalities

goal: to find where the graph is (+) or (-) depending on the inequality sign (remember to flip sign if multiply or divide by (-))

1. Get everything on one side and zero on the other
2. find LCD
3. Simplify the "everything" side into 1 fraction (not clearing fractions)
4. find x-intercepts - plot with open or closed holes depending on inequality signs
5. find restrictions (VA) - plot with open holes on line or if cross off undefined interval
6. use test point in intervals to find signs
7. Write answer in interval notation
\[
\frac{x - 2}{x} < \frac{x - 4}{x - 6} - \frac{x - 4}{x - 6} \quad \text{L.C.D: } x(x - 6)
\]

\[
\frac{(x - 6)x - 2}{x} - \frac{x - 4}{x - 6} \quad 0
\]

\[
\frac{(x - 6)(x - 2)}{x(x - 6)} - \frac{x(x - 4)}{x(x - 6)} \leq 0
\]

\[
x^2 - 8x + 12 - x^2 + 4x \leq 0
\]

\[
\frac{-4x + 12}{x(x - 6)} \leq 0
\]
\[
\frac{x - 5}{(2x - 3)(x + 2)} \geq 0
\]

\[
x \in [-2, \frac{3}{2}] \cup [5, \infty)
\]
\[
\frac{\sqrt{x-3}}{(2x+1)(x-4)} < 0
\]

\[
-\frac{1}{2} < x < 4
\]

\[
(3, 4)
\]