2.7 Solving Rational Equations

Rational Equation: an eq. (has an =) made up of 1 or more rational expressions

steps -
- find restrictions (why do I have restrictions?)
- Find the LCD
- Multiply each term in eq. by LCD to clear fractions
- solve the equation
- check for extraneous solutions
\[
\frac{x + 1}{3x - 6} = \frac{5x}{6} + \frac{1}{x - 2} \\
\text{Restrictions: } x \neq 2 \\
\text{LCM: } 6(x-2)
\]

\[
26(x-2) \left( \frac{x+1}{3(x-2)} \right) = 6(x-2) \left( \frac{5x}{6} \right) + 6(x-2) \left( \frac{1}{x-2} \right)
\]

\[
2(x+1) = 5x(x-2) + 6
\]

\[
2x+2 = 5x^2 - 10x + 6
\]

\[
0 = 5x^2 - 10x - 2x + 4
\]

\[
0 = 5x(x-2) - 2(x-2)
\]

\[
0 = (x-2)(5x-2)
\]

\[
\downarrow \quad \downarrow \\
2 \quad \frac{2}{5}
\]

\[
X = \frac{2}{5}
\]
\[
\frac{x - \frac{3x}{x + 2}}{1} = \frac{6}{x + 2}
\]

**R:** \(x \neq -2\)

**LCD:** \(x + 2\)

\[
(x + 2)x - (x + 2) \left( \frac{3x}{x + 2} \right) = (x + 2) \left( \frac{6}{x + 2} \right)
\]

\[
x(x + 2) - 3x = 6
\]

\[
x^2 + 2x - 3x - 6 = 0
\]

\[
x(x + 2) - 3(x + 2) = 0
\]

\[
(x + 2)(x - 3) = 0
\]

\[
\downarrow \quad \vee
\]

\[
-2 \quad 3
\]

\[
\boxed{x = 3}
\]
\[ \frac{7}{x} = 6 \]

\[ x(x) - 7 = 6x \]

\[ x^2 - 6x - 7 = 0 \]

\[ (x-7)(x+1) = 0 \]

\[ x = 7, -1 \]

\[ \text{L} \in \text{D} : x \neq 0 \]
\[ \frac{\frac{2}{x^3} \cdot (x-1) \cdot (x-3)}{x-1} + \frac{1}{x-3} = \frac{2(x-1)(x-3)}{x^2 - 4x + 3} \]

\[ 2x(x-3) + (x-1) = 2 \]

\[ 2x^2 - 6x + x - 1 = 2 \]

\[ \begin{align*} 
2x^2 - 5x - 1 & = \frac{2}{2} \\
2x^2 - 5x - 3 & \rightarrow \end{align*} \]

\[ 2x^2 - 6x + x - 3 \]

\[ 2x(x-3) + 3(x-3) = 0 \]

\[ (2x+1)(x-3) = 0 \]

\[ \begin{align*} 
& \downarrow \quad \downarrow \\
& x = -\frac{1}{2} \quad x = 3 \rightarrow \\
& \boxed{X = -\frac{1}{2}} \]
Mixture Problems

\[ \% = \frac{\text{part}}{\text{whole}} \]

\[ \text{concentration} = \frac{\text{acid}}{\text{total liquid}} \]

How much pure acid must be added to 70 mL of a 30% acid solution to reach mixture of 45% acid?

\[
(x+70) \cdot 0.45 = \frac{x + 21}{x + 70} \cdot 0.30 \cdot 70
\]

\[
0.45x + 31.5 = x + 21
\]

\[
0.45x + 10.5 = x
\]

\[
10.5 = 0.55x
\]

\[
19.09\text{mL} = x
\]
How much pure juice must be added to 100 mL of a 75% juice drink to reach a drink of 95% juice?
You want to put a rectangular pool in your backyard, you have an area of 200 square meters. To decrease costs, find the least perimeter of a rectangle with this area and the dimensions of the pool.

\[ \text{P} = 2l + 2w \]

\[ P(x) = 2x + 2\left(\frac{200}{x}\right) \]

\[ 2x + \frac{400}{x} = P(x) \]

\[ P : 56.56 \]

\[ w : 14.14 \]

\[ l : 14.14 \]