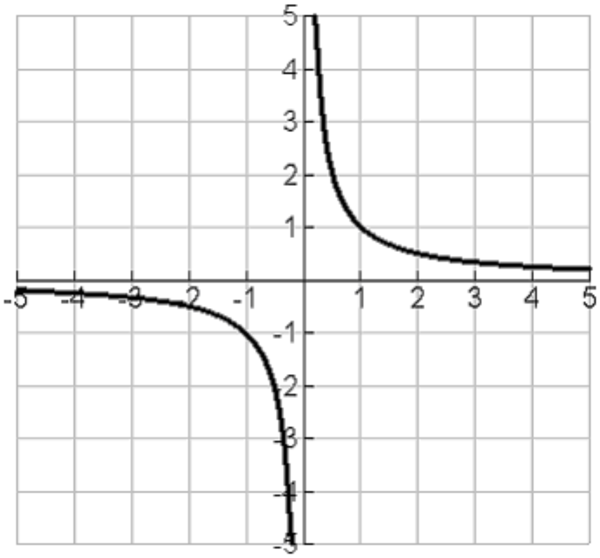
$$f(x) = \frac{1}{x}$$



Domain
Range
Continuous
Increasing
Decreasing
Constant
Left End
Right End
Symmetry
x-intercepts
y-intercepts
VA
HA
Bounded
Extrema

Transformations with rational functions:

If possible - rewrite in transformation form:

$$\frac{2}{x+3}$$

$$2\frac{1}{x+3}$$

vertical stretch by 2, left 3

If not - do the division:

$$\frac{3x-7}{x-2}$$

2	3	-7
		6
	3	-1

$$\boxed{3-\frac{1}{x-2}} \quad \text{or} \quad -\frac{1}{x-2}+3$$

up 3, right 2, flip over x-axis

Review

Find the vertical and horizontal asymptotes:

V: set den = 0 & solve   H: compare degrees

1. $y = \frac{3x-5}{x^2-4}$ $x^2-4=0$ $\sqrt{x^2}=\sqrt{4}$ $\boxed{x=\pm 2}$ $y=0$

2. $y = \frac{2x^3}{x-5}$ $x-5=0$ $\boxed{x=5}$ oblique

3. $y = \frac{5x}{x+2}$ $x+2=0$ $\boxed{x=-2}$ $y=\frac{5}{1}$ $\boxed{y=5}$

2.6 Graphs of Rational Functions

Goal - sketch a graph with x & y intercepts, vertical and horizontal asymptotes

x-intercepts: where cross x-axis

$y = 0$

numerator = 0 & solve

y-intercepts: where cross y-axis

$x = 0$

Plug in 0 for x & solve

Find the x & y intercepts:

1. $y = \dfrac{3x-5}{x^2-5x+6}$

$3x-5=0$
$+5 \quad +5$
$\dfrac{3x}{3} = \dfrac{5}{3}$
$x = 5/3$
$(5/3, 0)$

$y = \dfrac{3(0)-5}{0^2-5(0)+6}$
$y = \dfrac{-5}{6}$ $(0, -5/6)$

3. $y = \dfrac{x^2-2x+3}{x+2}$

$x^2-2x+3=0$
$x = \dfrac{2 \pm \sqrt{4-4(1)(3)}}{2(1)}$
no x-intercept
$y = 3/2$ $(0, 3/2)$

Asymptotes:

check for holes before VA!! (by reducing the fraction if possible)

vertical (VA): caused by dividing by 0

the graph approaches $-\infty \ \ or \ \ \infty$

on each side of the asymptote

to find the asymptote set den = 0 and solve

end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the num and den. if top heavy (OA):

bottom heavy (HA): y = 0

equal (HA): divide coefficients

oblique: (more later)

Find the holes, vertical and horizontal asymptotes:

1. $y = \dfrac{3x - 5}{x^2 - 5x + 6}$

$y = \dfrac{3x-5}{(x-3)(x-2)}$

VA: $x = 3, 2$

HA: $y = 0$

2. $y = \dfrac{4x}{2x^2 - 6x}$

$2x = 0$

$x = 0$

hole

$y = \dfrac{2\cancel{(2x)}}{\cancel{2x}(x-3)}$

$y = \dfrac{2}{x-3}$

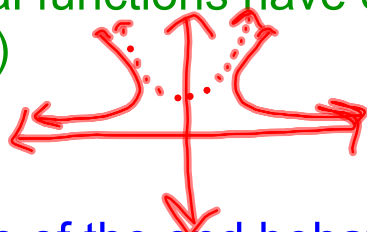
VA: $x = 3$

HA: $y = 0$

Oblique/Slant Asymptotes

#~~25~~
#26

top heavy rational functions have oblique asymptotes (end behavior models)



to find the degree of the end behavior model - divide the leading terms and reduce

the ends of $\dfrac{3x^5-4x^2+5}{2x^3-5x+4}$ will behave like $\dfrac{3x^5}{2x^3}=\boxed{\dfrac{3x^2}{2}}$

to find the actual asymptote: divide the fractions

Find the degree of the end behavior model & find the oblique asymptote:

3. $y = \dfrac{x^2 - 2x + 3}{x + 2}$

$\dfrac{x^2}{x} = x$ D: 1

$$\begin{array}{r|rrr} -2 & 1 & -2 & 3 \\ & \downarrow & -2 & 8 \\ \hline & 1 & -4 & 11 \end{array}$$

$\boxed{y = x - 4}$

4. $y = \dfrac{x^4 - 4x^2 + 4}{x - 2}$

$\dfrac{x^4}{x} = x^3$ D: 3

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -4 & 0 & 4 \\ & \downarrow & 2 & 4 & 0 & 0 \\ \hline & 1 & 2 & 0 & 0 & 4 \end{array}$$

$\boxed{y = x^3 + 2x^2}$

Limits

limits are about y behavior

#27

Limits about end behavior:
(related to horizontal asymptotes)

$$\lim_{x\to\infty} f(x)$$

this means the right end

$$\lim_{x\to-\infty} f(x)$$

this means the left end

Bottom Heavy

Top Heavy

Same (Equal)

there will be 4 possible answers: 0, ∞,-∞, #

1. $y = \dfrac{3x-5}{x^2-5x+6}$

$y=0$

$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

2. $y = \dfrac{4x}{2x-6}$

$y=\frac{4}{2}$ $y=2$

$\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to-\infty} f(x) = 2$

3. $y = \dfrac{x^2 - 2x + 3}{x + 2}$

$y = x - 4$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

4. $y = \dfrac{x^4 - 4x^2 + 4}{x - 2}$

$y = x^3 + 2x^2$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

Back of Notecard 30

At other places - usually around vertical asymptotes

$$\lim_{x\to 2^+} f(x)$$

when there is a number here - you look at that "x" location

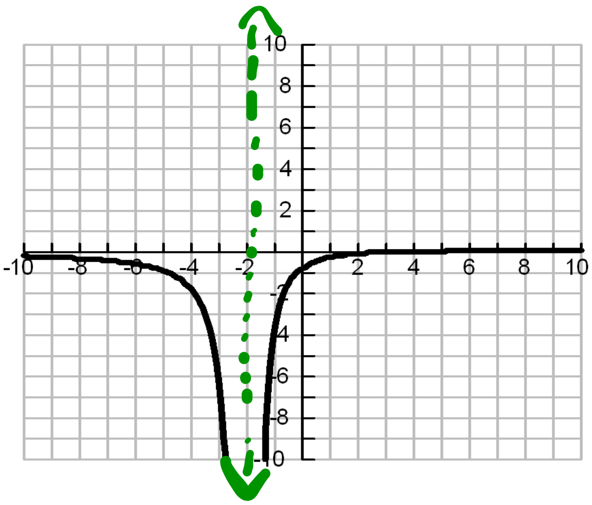
the "+" or "-" mean from the left or right of the #

$$\lim_{x\to 2^-} f(x)$$

power of the factor for the asymptote tells me if:

odd powered factor - graph is in opp directions on each side of asy.

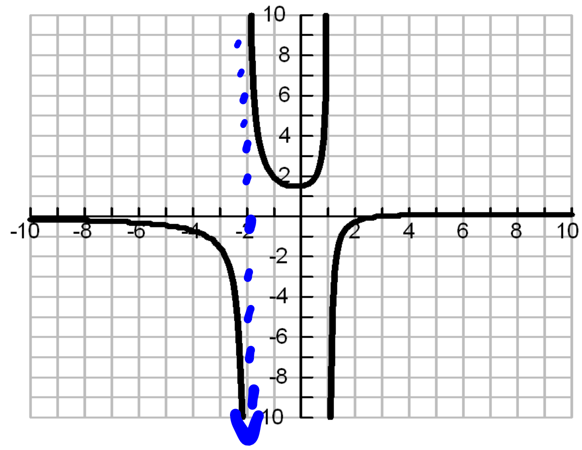
even powered factor - graph is the same direction on each side of asy.



$$y = \frac{x-3}{(x+2)^2}$$

$\lim_{x \to -2^+} f(x) = -\infty$

$\lim_{x \to -2^-} f(x) = -\infty$

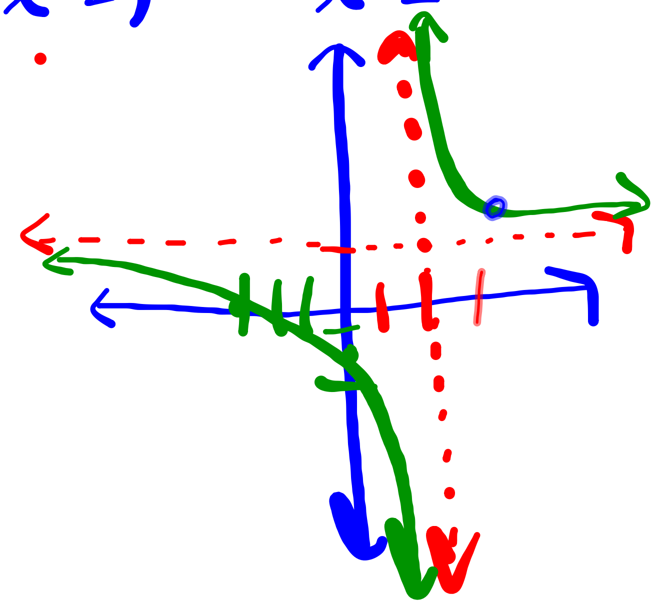


$$y = \frac{x-3}{(x-1)(x+2)}$$

$\lim_{x \to -2^+} f(x) = \infty$

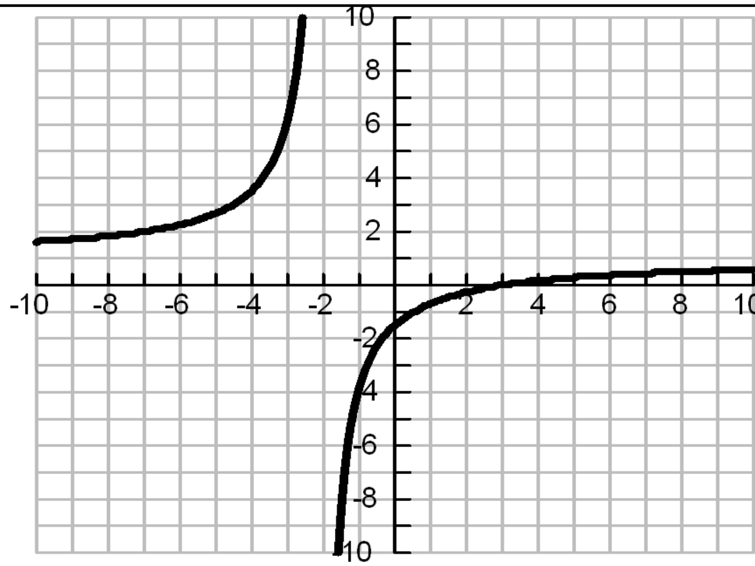
$\lim_{x \to -2^-} f(x) = -\infty$

$$y = \frac{x^2 - 9}{x^2 - 5x + 6} = \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x-2)} = \frac{x+3}{x-2}$$



$$\lim_{x\to 2^+} f(x) = \infty$$

$$\lim_{x\to 2^-} f(x) = -\infty$$



$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to -2^+} f(x) = -\infty$

$\lim_{x \to -2^-} f(x)$ ∞

Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{x^2-x-12}$$

$y=\dfrac{x-1}{(x-4)(x+3)}$

x-int: (1, 0)

y-int: (0, 1/12)

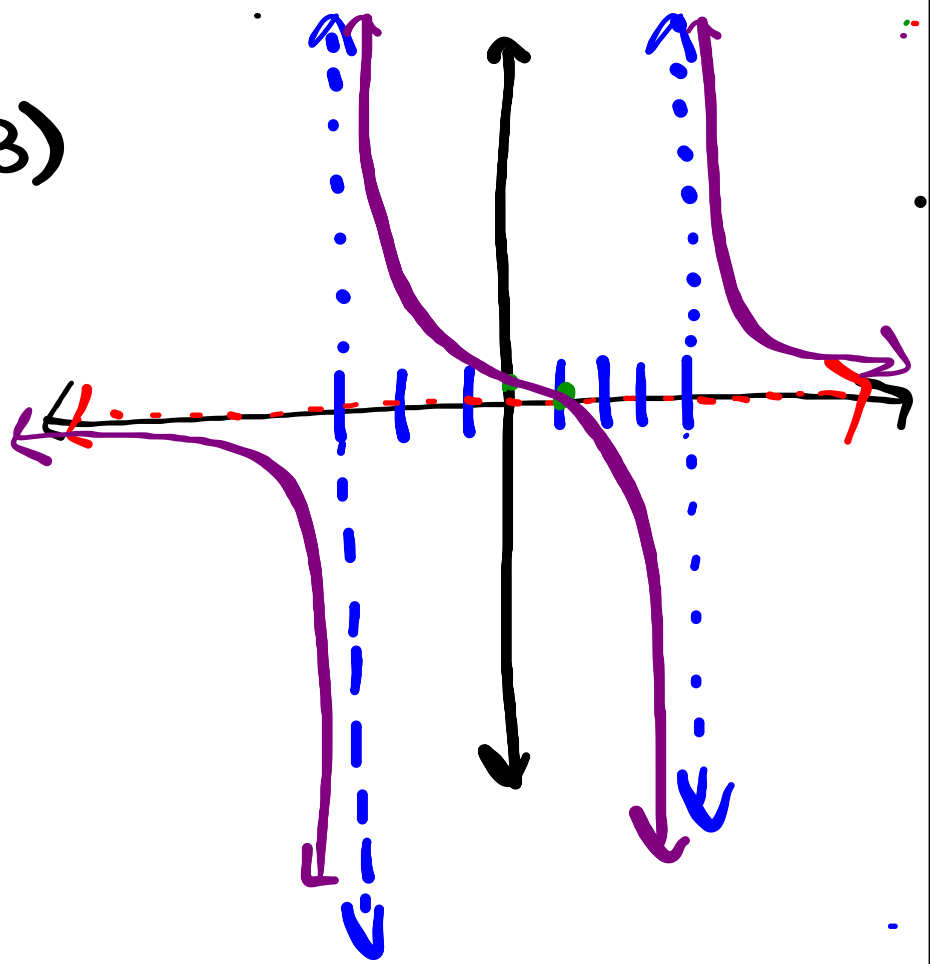
VA: $x=4,-3$

HA: $y=0$

Ends: $\to 0$

$x=4$ opp

$x=-3$ opp



x- Intercept
y-intercept
VA
HA (oblique)
end Behavior
Power Factors Denominator