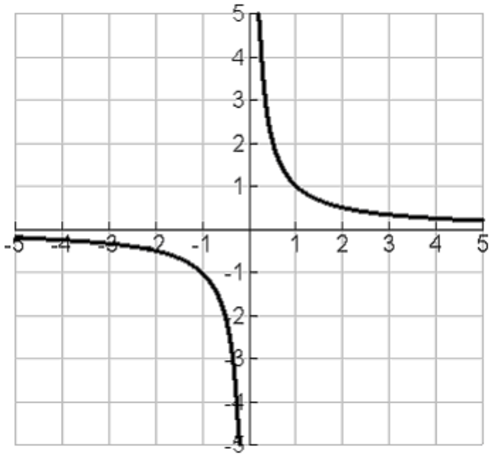
$$f(x) = \frac{1}{x}$$



Domain
Range
Continuous
Increasing
Decreasing
Constant
Left End
Right End
Symmetry
x-intercepts
y-intercepts
VA
HA
Bounded
Extrema

Transformations with rational functions:

If possible - rewrite in transformation form:

$$\frac{2}{x+3}$$

$$2\frac{1}{x+3}$$

vertical stretch by 2, left 3

If not - do the division:

$$\frac{3x-7}{x-2}$$

2	3	-7
		6
	3	-1

$$3-\frac{1}{x-2} \quad \text{or} \quad -\frac{1}{x-2}+3$$

up 3, right 2, flip over x-axis

Review

Find the vertical and horizontal asymptotes:

1. $y = \dfrac{3x-5}{x^2-4}$

2. $y = \dfrac{2x^3}{x-5}$

3. $y = \dfrac{5x}{x+2}$

2.6 Graphs of Rational Functions

Goal - sketch a graph with x & y intercepts,
vertical and horizontal asymptotes

x-intercepts:

y-intercepts:

Find the x & y intercepts:

1. $y = \dfrac{3x-5}{x^2-5x+6}$

3. $y = \dfrac{x^2-2x+3}{x+2}$

Asymptotes:

check for holes before VA!! (by reducing the fraction if possible)

vertical (VA): caused by dividing by 0

the graph approaches $-\infty \ or \ \infty$

on each side of the asymptote

to find the asymptote set den = 0 and solve

end behavior:(horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the num and den. if top heavy (OA):

bottom heavy (HA): y = 0

equal (HA): divide coefficients

oblique: (more later)

Find the holes, vertical and horizontal asymptotes:

1. $y = \dfrac{3x-5}{x^2-5x+6}$

2. $y = \dfrac{4x}{2x^2-6x}$

Oblique/Slant Asymptotes

#29

top heavy rational functions have oblique asymptotes (end behavior models)

to find the degree of the end behavior model - divide the leading terms and reduce

the ends of $\dfrac{3x^5-4x^2+5}{2x^3-5x+4}$ will behave like $\dfrac{3x^5}{2x^3} = \boxed{\dfrac{3x^2}{2}}$

to find the actual asymptote: divide the fractions

Find the degree of the end behavior model & find the oblique asymptote:

3. $y = \dfrac{x^2 - 2x + 3}{x + 2}$

4. $y = \dfrac{x^4 - 4x^2 + 4}{x - 2}$

Limits

#30

limits are about y behavior

Limits about end behavior:
(related to horizontal asymptotes)

$\lim_{x \to \infty} f(x)$ this means the right end

$\lim_{x \to -\infty} f(x)$ this means the left end

there will be 4 possible answers: 0, ∞,-∞, #

1. $y = \dfrac{3x-5}{x^2-5x+6}$

$\lim_{x\to\infty} f(x)$

$\lim_{x\to-\infty} f(x)$

2. $y = \dfrac{4x}{2x-6}$

$\lim_{x\to\infty} f(x)$

$\lim_{x\to-\infty} f(x)$

3. $y = \dfrac{x^2-2x+3}{x+2}$

$\lim_{x\to\infty} f(x)$

$\lim_{x\to-\infty} f(x)$

4. $y = \dfrac{x^4-4x^2+4}{x-2}$

$\lim_{x\to\infty} f(x)$

$\lim_{x\to-\infty} f(x)$

Back of Notecard 30

At other places - usually around vertical asymptotes

$$\lim_{x \to 2^+} f(x)$$

when there is a number here - you look at that "x" location

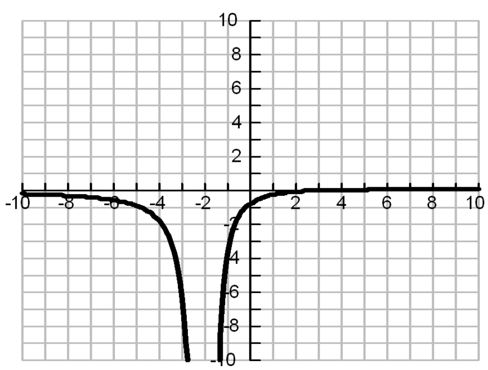
the "+" or "-" mean from the left or right of the #

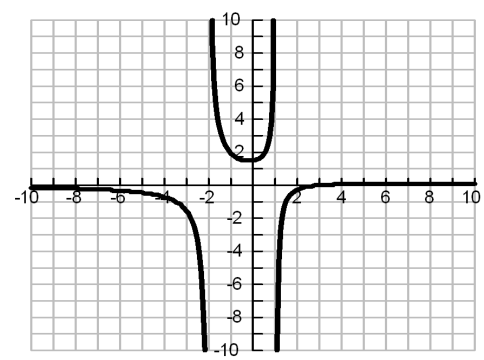
$$\lim_{x \to 2^-} f(x)$$

power of the factor for the asymptote tells me if:

odd powered factor - graph is in opp directions on each side of asy.

even powered factor - graph is the same direction on each side of asy.





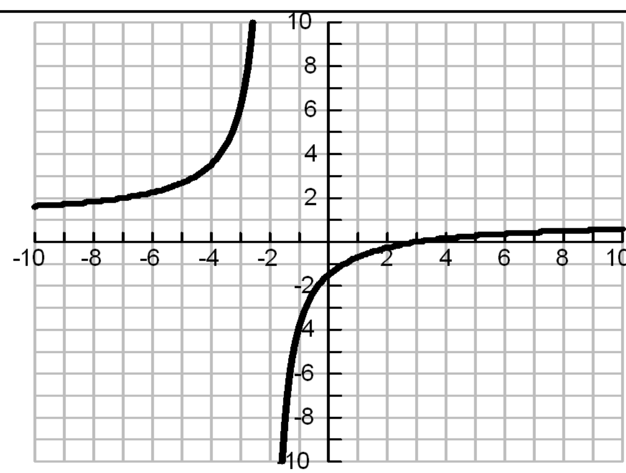
$$y = \frac{x-3}{(x+2)^2}$$

$$y = \frac{x-3}{(x-1)(x+2)}$$

$$y = \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$\lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} f(x)$$



$$\lim_{x \to \infty} f(x)$$

$$\lim_{x \to -\infty} f(x)$$

$$\lim_{x \to -2^+} f(x)$$

$$\lim_{x \to -2^-} f(x)$$

Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{x^2-x-12}$$

x-intercepts
y-intercepts
VA
HA
Limit at VA
Limit at VA
Domain
Range
Continuous
Increasing
Decreasing
End Behavior
Symmetry
Bounded
Extrema