Warm-Up

Write an equation with zeros of: 3, 5, -2

$y = (x-3)(x-5)(x+2)$ Factored

Write the polynomial in standard form:

$y = (x + 3)(x - 2)$

$y = x^2 + x - 6$ Standard

Write an equation of minimum degree with given zeros and multiplicities:

1 with multi of 3

-3 with multi of 2

0 with multi of 1

2.5 Complex zeros & Fundamental Thm of Algebra

Top Half of Card 27 #27

Fundamental Thm of Alg: an nth degree polynomial will have n zeros

(may be a combination of real and complex & some zeros may be repeated)

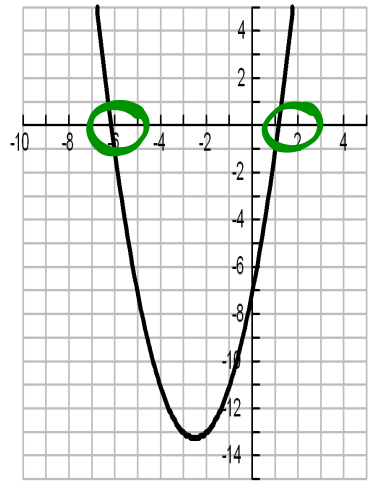
Odd functions will always have at least one real zero - why??

Complex Conjugates: complex factors come in conjugate pairs
(if 3i is a zero, - 3i is also)

Top Left side of back of #27

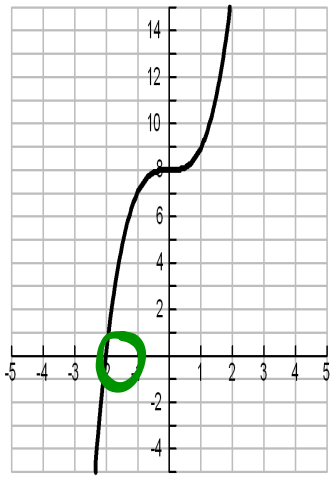
$x^2 + 5x - 7$

C:2 R:2



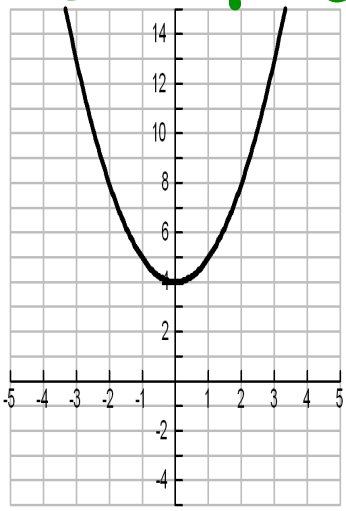
$x^3 + 8$

C:3 R:1



$x^2 + 4$

C:2 R:0



How many real and complex zeros does each have??

Write a polynomial function of minimum degree with the following zeros:

4, 7, 2i, -2i

$y = (x-4)(x-7)(x-2i)(x+2i)$

Bottom Half of Card #27

Linear Factorization Thm: a polynomial of nth degree has n linear factors

(some factors may be complex)

$$x^4 - 6x^3 + 10x^2 - 6x + 9$$

$$(x-3)(x+3)(x-i)(x+i)$$

Find all zeros and write a linear factorization of the following polynomial:

$x^4 + x^3 + 5x^2 - x - 6$

$\pm 1 \pm 2 \pm 3 \pm 6$

± 1

± 1

	1	1	5	-1	-6
-1		-1	0	-5	6
	1	0	5	-6	0
1		1	1	6	
	1	1	6	0	

$(x+1)$ $(x-1)$

$x^2 + x + 6 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(6)}}{2}$

$x = \frac{-1 \pm \sqrt{1 - 24}}{2}$

$x = \frac{-1 \pm \sqrt{-23}}{2}$

$= \left\{ \frac{-1 \pm i\sqrt{23}}{2}, 1, -1 \right\}$ zeros

Linear factorization $(x+1)(x-1)\left(x - \left(\frac{-1+i\sqrt{23}}{2}\right)\right)\left(x - \left(\frac{-1-i\sqrt{23}}{2}\right)\right)$

$x + \frac{1 - i\sqrt{23}}{2}$

$x + \frac{1 + i\sqrt{23}}{2}$

Write the following polynomial in standard form:

$$y = (x + 3i)(x - 3i)$$

$x^2 - 3ix + 3ix - 9(i^2)$

$x^2 + 9$ $(-9)(1)$

Write a polynomial function of minimum degree with the following zeros in standard form:

$-4, 2+3i, 2-3i$

$2+3i = \Delta$

$2-3i = \heartsuit$

$y = (x+4)(x-(2+3i))(x-(2-3i))$

$y = (x+4)(x-\Delta)(x-\heartsuit)$

$y = (x+4)(x^2 - x\heartsuit - x\Delta + \heartsuit\Delta)$

$y = (x+4)(x^2 - x(2-3i) - x(2+3i) + (2-3i)(2+3i))$

$y = (x+4)(x^2 - 2x + 3ix - 2x - 3ix + 4 - 9i^2)$

$+13 \qquad +9$

$y = (x+4)(x^2 - 4x + 13)$

$y = x^3 - 4x^2 + 13x + 4x^2 - 16x + 52$

$\boxed{y = x^3 - 3x + 52}$

Write an equation of minimum degree with given zeros and multiplicities:

3 with multi of 2
5 + i with multi of 1

Use the given zero to find the remaining zeros and write a linear factorization:

$3-2i;\ x^4-6x^3+11x^2+12x-26$

$(3-2i)(-3-2i) = -9+4i^2 = -9-4 = -13$

$3+2i$

$(3-2i)(6+4i) = 18-8i^2 = 18+8$

$3-2i$	1	-6	11	12	-26
		$3-2i$	-13	$-6+4i$	26
$3+2i$	1	$-3-2i$	-2	$6+4i$	0
		$3+2i$	0	$-6-4i$	
	1	0	-2	0	

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

Zeros: $3-2i, 3+2i, \sqrt{2}, -\sqrt{2}$

$f(x) = (x-(3-2i))(x-(3+2i))(x-\sqrt{2})(x+\sqrt{2})$

Every Polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors.

Irreducible Quadratic: Quadratic with real coefficients but no real zeros

$$f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$$