Warm-Up

Write an equation with zeros of: 3, 5, -2

$f(x) = (x-3)(x-5)(x+2)$

Write the polynomial in standard form:

$y = (x + 3)(x - 2)$

$x^2 + x - 6$

Write an equation of minimum degree with given zeros and multiplicities:

1 with multi of 3

-3 with multi of 2

0 with multi of 1

$x(x+3)^2(x-1)^3$

2.5 Complex zeros & Fundamental Thm of Algebra

Top Half of Card 27

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Fundamental Thm of Alg: an nth degree polynomial will have n zeros

(may be a combination of real and complex & some zeros may be repeated)

Odd functions will always have at least one real zero -why??

Complex Conjugates: complex factors come in conjugate pairs
(if 3i is a zero, - 3i is also)

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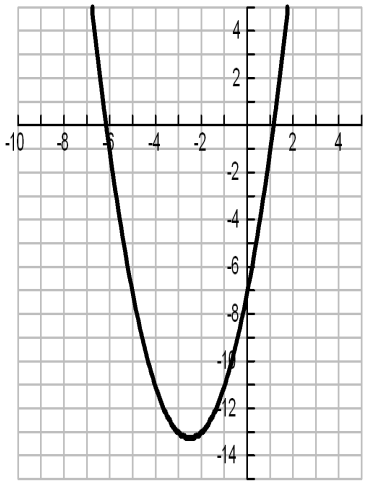
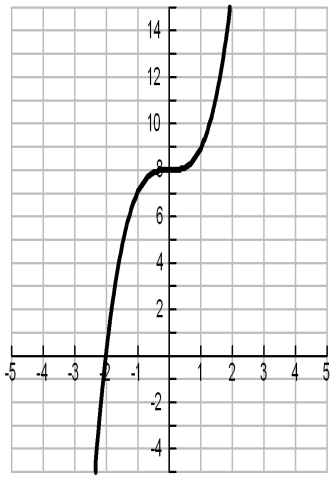
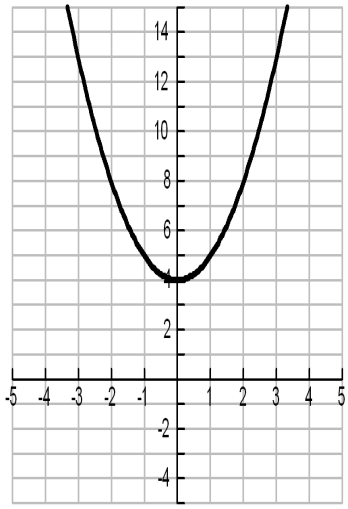
$x^2 + 5x - 7$



$x^3 + 8$

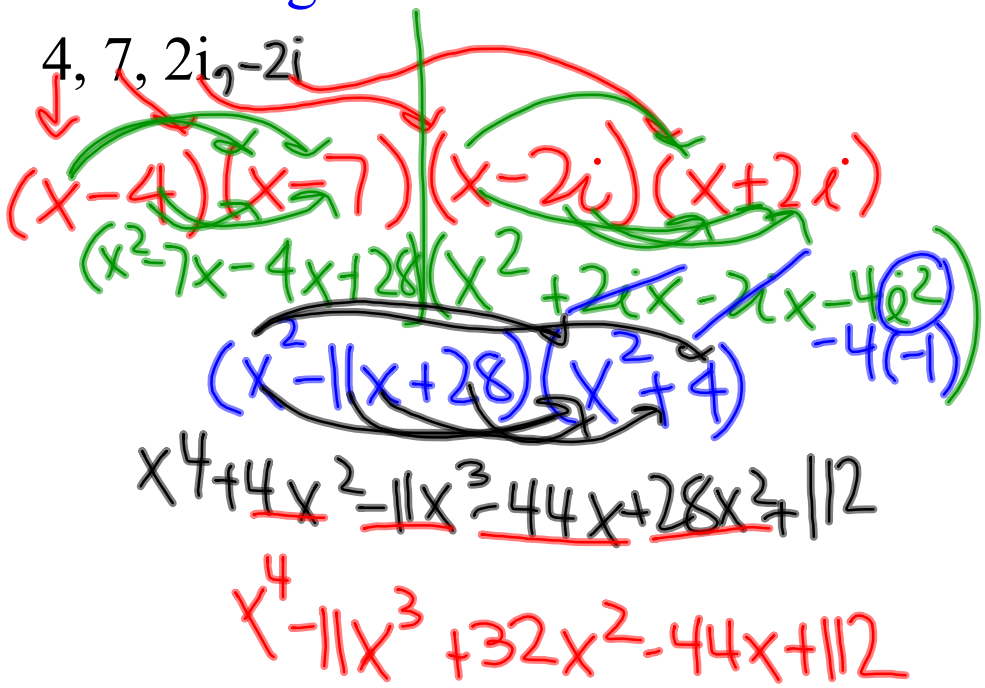


$x^2 + 4$



How many real and complex zeros does each have??

Write a polynomial function of minimum degree with the following zeros:



Bottom Half of Card #27

Linear Factorization Thm: a polynomial of nth degree has n linear factors

(some factors may be complex)

$$x^4 - 6x^3 + 10x^2 - 6x + 9$$

$$(x-3)(x+3)(x-i)(x+i)$$

Find all zeros and write a linear factorization of the following polynomial:

$$x^4 + x^3 + 5x^2 - x - 6$$

$\frac{\pm 1 \pm 6 \pm 2 \pm 3}{\pm 1} = 1, -1$

	1	1	5	−1	−6	
1		↓ 1	2	7	6	(x−1)
−1		1	2	7	6	☺ (x+1)
		↓ 1	−1	−1	−6	
		1	1	6	☺	

$a=1$ $b=1$ $c=6$ $x^2+x+6=0$

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(6)}}{2(1)} = \frac{-1 \pm \sqrt{1-24}}{2} = \frac{-1 \pm \sqrt{-23}}{2}$$

ZEROS: $\frac{-1 \pm \sqrt{23}i}{2}, \pm 1$

Linear Factorization: $(x-1)(x+1)\left(x-\left(\frac{-1+\sqrt{23}i}{2}\right)\right)\left(x-\left(\frac{-1-\sqrt{23}i}{2}\right)\right)$

Write the following polynomial in standard form:

$$y = (x + 3i)(x - 3i)$$

Write a polynomial function of minimum degree with the following zeros in standard form:

$-4, 2+3i, 2-3i$

$2+3i = q$

$2-3i = h$

$(x+4)(x-(2+3i))(x-(2-3i))$

$(x+4)(x-q)(x-h)$

$(x+4)(x^2-xh-xq+qh)$

$(x+4)(x^2-x(2-3i)-x(2+3i)+(2-3i)(2+3i))$

$-9i^2 = 9$

$(x+4)(x^2-2x+3ix-2x-3ix+4+9)$

$(x+4)(x^2-4x+13)$

$x^3-4x^2+13x+4x^2-16x+52$

$x^3-3x+52$

Write an equation of minimum degree with given zeros and multiplicities:

3 with multi of 2
5 + i with multi of 1

Use the given zero to find the remaining zeros and write a linear factorization:

$$3-2i;\ x^4-6x^3+11x^2+12x-26$$

$3+2i$

$(3+2i)(-3+2i) = -9+4i^2 = -9-4 = -13$

$(3+2i)(6-4i) = 18-8i^2 = 26$

$3+2i$	1	-6	11	12	-26
		$3+2i$	-13	$-6-4i$	26
$3-2i$	1	$-3+2i$	-2	$6-4i$	0
		$3-2i$	0	$-6+4i$	
	1	0	-2	0	

$x^2-2=0 \Rightarrow x^2=2 \Rightarrow x=\pm\sqrt{2}$

ZEROS: $3-2i, 3+2i, \sqrt{2}, -\sqrt{2}$

Linear Factorization: $(x-(3-2i))(x-(3+2i))(x-\sqrt{2})(x+\sqrt{2})$

$(x-3+2i)(x-3-2i)$

Every Polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors.

Irreducible Quadratic: Quadratic with real coefficients but no real zeros

$$f(x)=3x^5-2x^4+6x^3-4x^2-24x+16$$

$\pm1,\pm16,\pm4,\pm8,\pm2$ / $\pm1,\pm3$ $\quad 2/3$

$2/3$ | 3 | −2 | 6 | −4 | −24 | 16

	3	0	6	0	−24	☺
↓		2	0	4	0	−16

$3x^4+6x^2-24$

$3(x^4+2x^2-8)$

$3(x^2+4)(x^2-2)(x-2/3)$

irreducible

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

$3(x-2/3)(x+\sqrt{2})(x-\sqrt{2})(x^2+4)$