2.4 Real Zeros for a Polynomial

Divide: 50213 $\div$ 7

$$
\begin{array}{r}
7173\ \tfrac{2}{7} \\
7\overline{)50213} \\
\underline{-49} \\
12 \\
\underline{-7} \\
51 \\
\underline{-49} \\
23 \\
\underline{-21} \\
2
\end{array}
$$

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Dividing Polynomials - Long Division 17 #~~28~~

Steps: 1. Write as a division problem w/ dividends & divisor in descending order, leaving spaces for missing terms in the dividend (0x)

2. Divide leading terms and write the result above the 1st term in the dividend $\begin{array}{l} \ \ x \\ \overline{)\quad} \end{array}$

3. Multiply the result from #2 by the divisor & write the product under the dividend

4. Put ( ) around result from #3, distribute the subtraction sign & then add

5. Bring down remaining terms & repeat until there are no remaining terms in the dividend

6. Answer can be written in several ways - see back

Divide using long division:

1. $(5x^3 + 3x - 8) \div (x - 1)$

$$
\begin{array}{r}
5x^2 + 5x + 8 \\
x-1 \overline{) \; 5x^3 + 0x^2 + 3x - 8} \\
-\; 5x^3 - 5x^2 \qquad\qquad \\
\hline
0 + 5x^2 + 3x \qquad \\
-\; 5x^2 - 5x \qquad \\
\hline
0 \quad + 8x - 8 \\
-\; 8x - 8 \\
\hline
0 + \text{☺}
\end{array}
$$

polynomial form or fraction form

$f(x) = (x-1)(5x^2 + 5x + 8) +$ ☺

$$\frac{f(x)}{x-1} = 5x^2 + 5x + 8 + \frac{0}{x-1}$$

Answer Forms

[redacted]

Polynomial form: $$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}} \cdot \underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}$$

When r(x)=0 we say that d(x) divides evenly into f(x)

Fraction Form:

$$\frac{f(x)}{d(x)} = \frac{d(x) \cdot q(x) + r(x)}{d(x)} \qquad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

$$(x^4 - 2x^3 + 3x^2 - 4x + 6) \div (2x + x^2 + 1)$$

$$
\begin{array}{r|l}
 & x^2 - 4x + 10 \\
\hline
x^2+2x+1 & x^4 - 2x^3 + 3x^2 - 4x + 6 \\
- & \underline{x^4 + 2x^3 + x^2} \\
 & 0 - 4x^3 + 2x^2 - 4x \\
- & \underline{-4x^3 - 8x^2 - 4x} \\
 & 0 + 10x^2 + 0x + 6 \\
- & \underline{10x^2 + 20x + 10} \\
 & 0 \quad \boxed{-20x - 4}
\end{array}
$$

$$f(x) = (x^2+2x+1)(x^2-4x+10) - 20x - 4$$

$$\frac{f(x)}{x^2+2x+1} = x^2 - 4x + 10 + \frac{-20x-4}{x^2+2x+1}$$

Dividing Polynomials - Synthetic division: ~~#2~~4

Can only be used to divide by a linear function

steps:

1. Write the terms of the dividend in descending order. Write the coeff. of the dividend in the first row using zeros for any missing terms not found in the dividend.
2. Write the zero, r, of the divisor (x-r),in the box.
3. Drop the 1st coeff. to the last row.
4. Multiply 1st coeff. by r & put product under the 2nd coeff.
5. Add product from #4 to 2nd coeff. & write the sum in the last row.
6. Repeat #4 & #5 until all coeff. have been used.
7. Write answer by putting variables behind the #'s in the last row. Start with 1 degree less than the dividend polynomial.

$$\frac{x^3 - 5x^2 + 3x - 2}{x+1}$$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 3 & -2 \\ & \downarrow & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & -11 \end{array}$$

$\leftarrow$ Remainder

$x^2 - 6x + 9 - \frac{11}{x+1}$

$$\frac{x^4 - 3x^2 - 2}{x - 5}$$

$$\begin{array}{r|rrrrr} 5 & 1 & 0 & -3 & 0 & -2 \\ & & 5 & 25 & 110 & 550 \\ \hline & 1 & 5 & 22 & 110 & 548 \end{array}$$

$$x^3 + 5x^2 + 22x + 110 + \frac{548}{x-5}$$

$\left(x^3 - x^2 + 2x - 7\right) \div \left(2x - 1\right)$

to get (x-r): divide num. & den. by 2 to get $(x-\frac{1}{2})$

What is the significance of the answers??

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Remainder Theorem & Factor Theorem

Remainder Theorem: $f(k)$ = remainder

this means - evaluate the function for the value of the suspected zero (plug it in for x)

Factor Theorem: if the remainder is 0 then you have found a root!!! ($f(k) = 0$)

Use the Remainder & Factor Thm. to find if the first polynomial is a factor of the second: (there are 2 ways)

$x-3$, x^3-x^2-x-15

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -1 & -15 \\ & & 3 & 6 & 15 \\ \hline & 1 & 2 & 5 & ☺ \end{array}$$

$f(3) = (3)^3 - (3)^2 - (3) - 15$

$= 27 - 9 - 3 - 15$

$= 27 - 27$

$=$ ☺

Rational Root Theorem: 21 ~~#138~~

if all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

or $x = \pm \dfrac{p}{q}$ when

p = factors of constant

q = factors of leading coefficient

Types of Zeros

Rational Zeros:

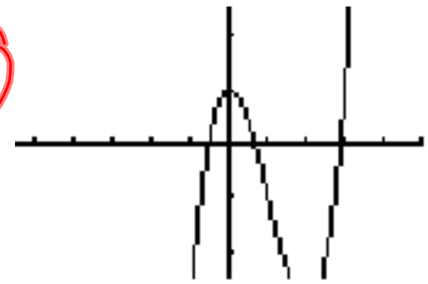
$$f(x) = 4x^2 - 9 = (2x+3)(2x-3)$$

Irrational Zeros:

$$f(x) = x^2 - 2 = (x+\sqrt{2})(x-\sqrt{2})$$

Find all possible rational roots & determine if any are zeros

$$f(x) = x^3 - 3x^2 + 1$$



$$\frac{\pm 1}{\pm 1};\ \cancel{1},\ \cancel{-1}$$

$$f(1) = 1^3 - 3(1^2) + 1 = 1 - 3 + 1 = -1$$

$$f(-1) = (-1)^3 - 3(-1^2) + 1 = -1 - 3 + 1 = -3$$

Find all possible rational roots & determine if any are zeros

$$2x^4 - 7x^3 - 8x^2 + 14x + 8$$

$$\frac{\pm 2 \pm 1 \pm 4 \pm 8}{\pm 2 \pm 1};\ \pm 1, \pm 2, \pm \tfrac{1}{2}, \pm 4, \pm 8$$

$-\frac{1}{2}, 4$

$$\begin{array}{r|rrrrr}
4 & 2 & -7 & -8 & 14 & 8 \\
 & \downarrow & 8 & 4 & -16 & -8 \\
\hline
-\tfrac{1}{2} & 2 & 1 & -4 & -2 & \\
 & \downarrow & -1 & 0 & +2 & \\
\hline
 & 2 & 0 & -4 & &
\end{array}$$

$$(x-4)(2x^3+x^2-4x-2)$$

$$(x-4)(x+\tfrac{1}{2})(2x^2-4)$$

$$(x-4)(x+\tfrac{1}{2})(x+\sqrt{2})(x-\sqrt{2})$$

$$2x^2-4=0$$
$$+4 \quad +4$$
$$\frac{2x^2}{2}=\frac{4}{2}$$
$$\sqrt{x^2}=\sqrt{2}$$

$$x=\pm\sqrt{2}$$

$4, -\frac{1}{2}, \sqrt{2}, -\sqrt{2}$

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Upper and Lower Bound Test for REAL ZEROS

#27

For a function f(x) with a degree $n \geq 1$ with a positive leading coefficient. We divide f(x) by x-k using synthetic division.

If $k \geq 0$ and every number in the last line is NON NEGATIVE (+ or 0) then k is an upper bound for the real zeros of f.

If $k \leq 0$ and the numbers in the last line are alternately nonnegative and nonpositive then k is an lower bound for the real zeros of f.

Prove all the zeros of $2x^4 - 7x^3 - 8x^2 + 14x + 8$ must lie in the interval [-2,5]

5	2	-7	-8	14	8
	↓	10	15	35	245
	2	3	7	49	253

upper

+

0 →

-2	2	-7	-8	14	8
	↓	-4	22	-28	28
	2	-11	14	-14	36

Lower Bound

alternates

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Find all possible rational roots & determine if any are zeros

$$2x^4 - 7x^3 - 8x^2 + 14x + 8$$