2.3 Polynomial Functions

Polynomial Functions

Standard Form of a polynomial function:

$$\boxed{a_n x^n} + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + \ldots\boxed{a_0}$$

term: each part of the polynomial - separated by + or (-)
leading term: term with the highest power or 1st term if written in standard form (poly. must be multiplied out to find this)
coefficient: number in front of the variable
constant: number w/o a variable

Degree: highest power found in any given term

What is the degree of:

$$y = -89x^{6} + 3x^{5} + 2x^{3} - 7x + 2$$

D:6

$$y = (x-5)^{2}(x+2)^{3}$$

D:5

$$y = x^{2}(2x-3)(x+5)^{3}(x+1)^{2}$$

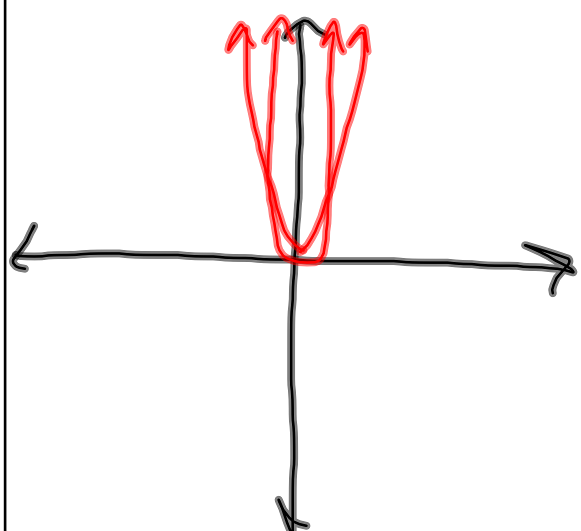
D:8

$$y = \frac{5}{3}x^{5} + 3x^{3} - 7x^{2} + x - 12$$

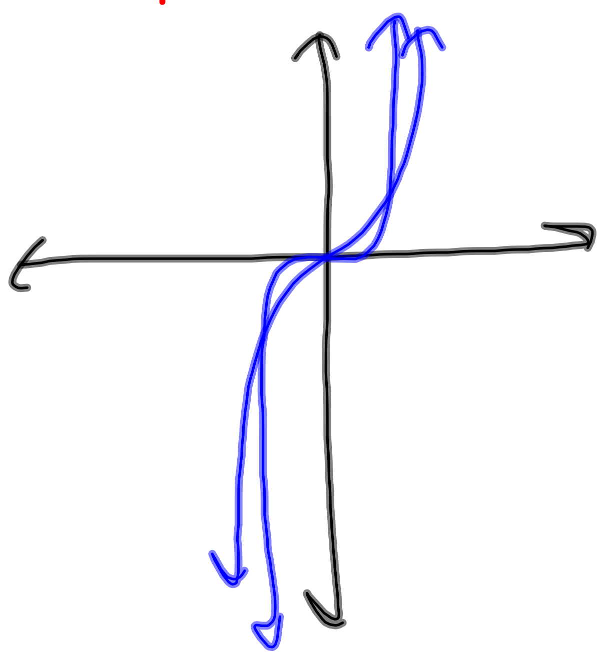
D:5

Transformations

Graph x^2 and x^4



Graph x^3 and x^5



Describe the Transformation, sketch the graph compute y-intercept

$$g(x)=4(x+1)^3$$

$y=4(0+1)^3$

$y=4(1)$

$y=4$

$(0,4)$

$$h(x)=-(x-2)^4+5$$

$y=-(0-2)^4+5$

$y=-(-2)^4+5$

$y=-16+5$

$y=-11$

$(0,-11)$

Graphing Combination Functions

$$f(x) = x^3 + x$$

1. Factor
2. Find Zeros

$$g(x) = x^3 - x$$

What happens if we make the leading coefficient (-)?

End Behavior

Graph $f(x) = x^3 - 4x^2 - 5x - 3$ $g(x) = x^3$

What happens as we continue to zoom out?

Where is each end going?

End Behavior (polynomial)

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End Behavior is determined by the degree of the polynomial and the coefficient of the leading term. The mathematical notation is written using limits.

$$\lim_{x \to -\infty} f(x) =$$

left end

$$\lim_{x \to \infty} f(x) =$$

right end

Odd Degree: the left & right ends go in opp. directions

(+) coeff.

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

(-) coeff.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$

Even Degree: both ends go in the same direction

(+) coeff. both up

$$\lim_{x \to \infty} f(x) = \infty$$

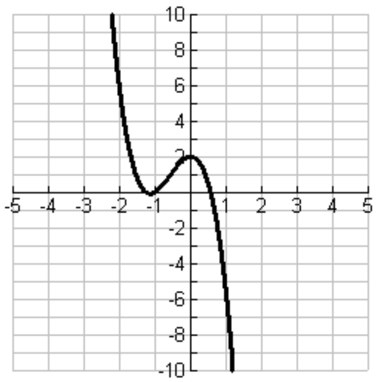
$$\lim_{x \to -\infty} f(x) = \infty$$

(-) coeff. both down

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

Name the degree & the sign of the coefficient of the leading term based on the end behavior:



ODD

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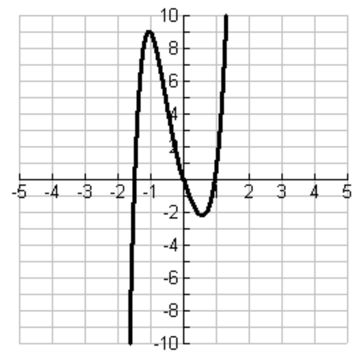

ODD

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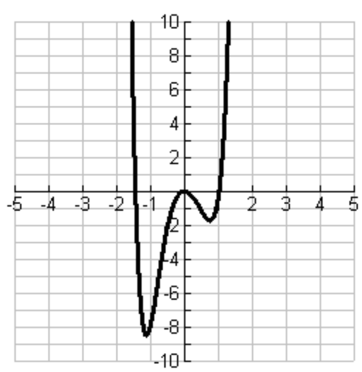
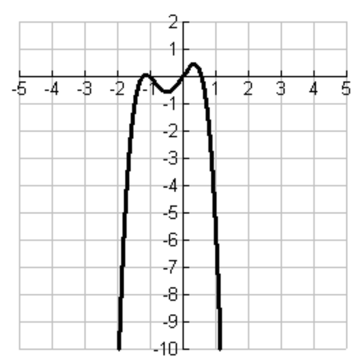


EVEN

+



EVEN

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Graph and decide end behavior

$a) f(x) = x^3 + 2x^2 - 11x - 12$

$\lim_{x \to \infty} f(x) = \infty$ $\qquad \lim_{x \to -\infty} f(x) = -\infty$

$b) g(x) = 2x^4 + 2x^3 - 22x^2 - 18x - 35$

$\lim_{x \to \infty} g(x) = \infty$ $\qquad \lim_{x \to -\infty} g(x) = \infty$

Zeros(roots) and Mulitiplicity

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Zeros: solutions for x when y = 0

can be found in the **factors** $(x - a)$ of the polynomial.

How do we find the zeros??

- factor
- quadratic formula
- use the calculator

What are the differences between factors and zeros???

$(x-a)$ FACTOR

a ZERO

Find the zeros of: $y = 3x^3 - 5x^2 + 2x$

$a \cdot c = 6$

$x(3x^2 - 5x + 2)$

$x(\underbrace{3x^2 - 3x}\underbrace{-2x + 2})$

$x(\underbrace{3x(x-1) - 2(x-1)})$

$x(x-1)(3x-2) = 0$

$\boxed{0, 1, 2/3}$

Given the zeros, write a polynomial equation of given degree:

degree 5, zeros: 0, 2, -5

$x^2(x-2)^2(x+5)$

degree 4, zeros: -2, 2

$(x+2)^2(x-2)^2$

degree 4, zeros: -5, 0, 5

$x^2(x+5)(x-5)$

Practice:

Find the zeros of:

$$y = x^4 - 8x^2 - 9$$

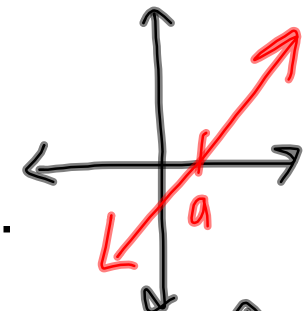
multiplicity

18 ~~22~~

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the multiplicity.)

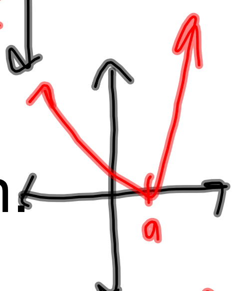
Straight intersection:

$(x - a)^1$ The power of the zero is 1.



Tangent intersection : Bounces

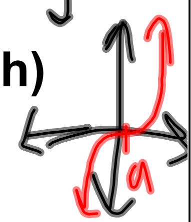
$(x - a)^{even}$ The power of the zero is even.



Kisses

Inflection intersection: **(like a slide through)**

$(x - a)^{odd}$ The power of the zero is odd.



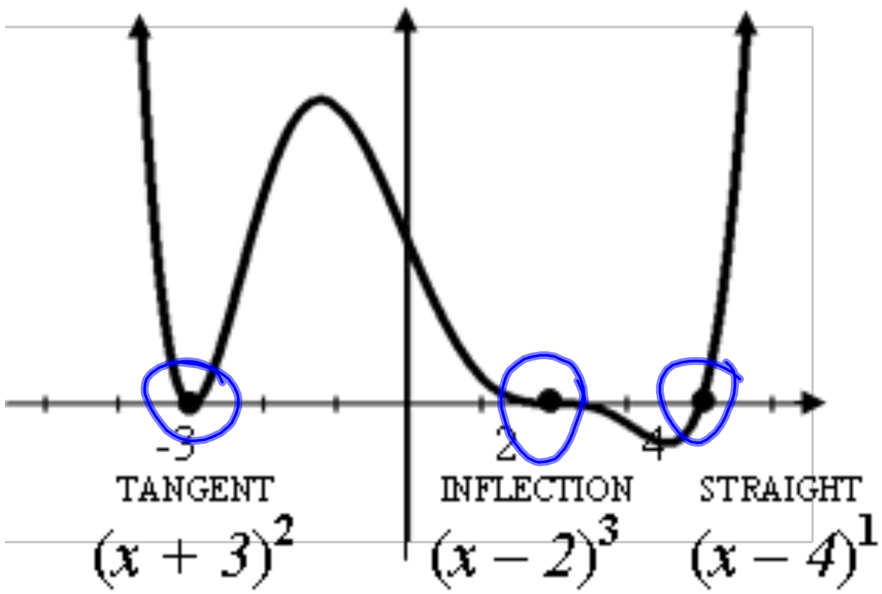
$$y = (x + 3)^2 (x - 2)^3 (x - 4)$$

-3 : 2 2 : 3 4 : 1

What are the zeros??

What is the mulitplicity (power) of the zero??

How will it intersect the x-axis??



Practice:

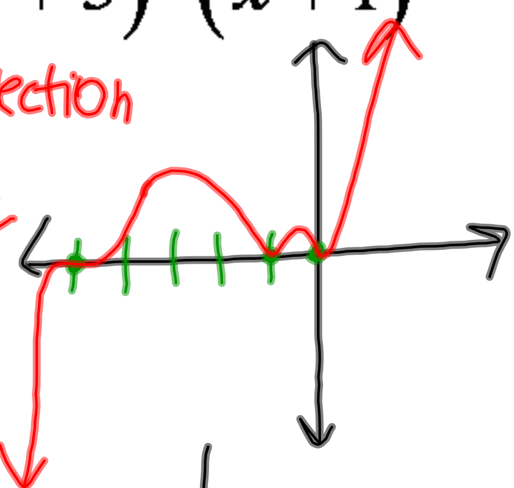
Sketch the graph of: $y = x^2(x+5)^3(x+1)^2$

0: 2 :Tangent/Bounce

−5: 3 :Slide/kisses/inflection

−1: 2 :Tangent/Bounce

D:7 ODD+

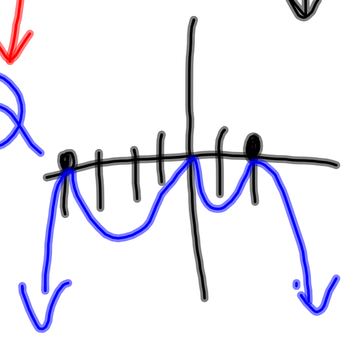


$y = -5x^2(x-2)^2(x+4)^2$

0:2 2:2 −4:2

tan/bounce

6 even −



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