# Step Functions

Objectives:

I can write and graph step function problem situations.
I can analyze the graphs of step functions.

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In 2004, Georgia had 6 income tax brackets. The tax rate on every dollar of income was:

- 1% for incomes more than $0 and up to and including $750 (.01)
- 2% for incomes more than $750 and up to and including $2250 (.02)
- 3% for incomes more than $2250 and up to and including $3750 (.03)
- 4% for incomes more than $3750 and up to and including $5250 (.04)
- 5% for incomes more than $5250 and up to and including $7000 (.05)
- 6% for incomes more than $7000 (.06)

1. Write a piecewise function f(x) for the tax paid in Georgia for income x.

$$f(x)\begin{cases} .01x & 0<x\le 750 \\ .02x & 750<x\le 2250 \\ .03x & 2250<x\le 3750 \\ .04x & 3750<x\le 5250 \\ .05x & 5250<x\le 7000 \\ .06x & 7000<x \quad x>7000 \end{cases}$$

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2. Calculate the amount of tax paid with an income of:

a. $750 $\quad .01(750) = \$7.50$

b. $751 $\quad .02(751) = \$15.04$

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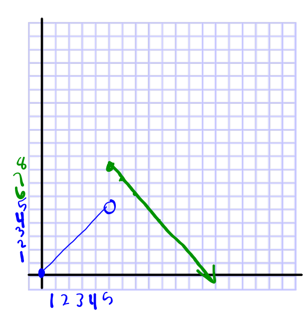
Let's consider the linear piecewise function, f(x), and how to create a graph.

y-value x-value

$$f(x)=\begin{cases} x, & 0\le x<5 \\ -x+13, & x\ge 5 \end{cases}$$

(0,0)

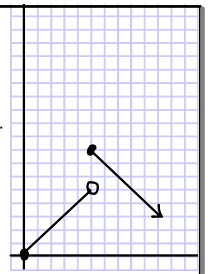
(5,8)



3. Analyze the linear piecewise function in the worked example.

   a. Notice that the function is not continuous. At what x-value is there a break in the graph? Why do you think that break occurs?

   5, changed equations



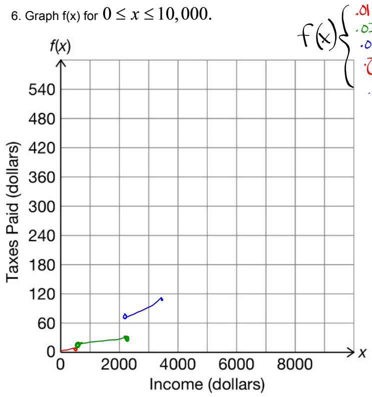
5. Analyze the function you wrote to represent the tax brackets in Georgia from Question 1.

   a. Would the lines that represent each piece of the function on the graph be connected or contain breaks? Explain.

   Breaks, changed equations

   b. Describe the endpoints of the line representing each interval. Explain.

6. Graph f(x) for $0 \le x \le 10,000$.




$$f(x)\begin{cases} .01x & 0 \le x \le 750 \\ .02x & 750 < x \le 2250 \\ .03x & 2250 < x \le 3750 \\ .04x & 3750 < x \le 5250 \\ .05x & 5250 < x \le 7000 \\ .06x & 7000 < x \\ & x > 7000 \end{cases}$$

7. Describe the rate of change when:

   a. $0 \le x \le 750$

   $\frac{1}{100}$ slowly increasing

   b. $750 < x \le 1000$

   $\frac{2}{100}$ faster than a but not too fast

8. Calculate the amount of tax paid on an income of:

a. $2250 = $45 in taxes per month

b. $2251 = $67.53 per month

c. $7000 = $350

d. $7001 = $420

No good

9. Describe the method you used to calculate the amount of tax for year income.

## Problem 2: Taxi Fares

In 2006, the rate for a taxi ride in Macon, Georgia, was $1.20 for the first mile or part of a mile, and $1.20 for each additional mile or part of a mile.

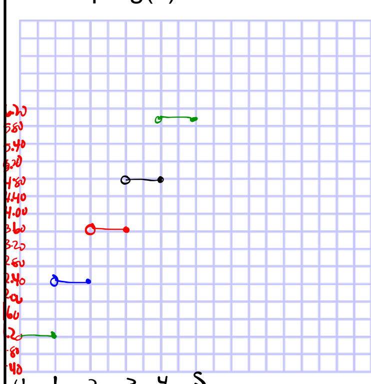
1. Define a piecewise function, g(x), for the cost of a taxi ride up to 5 miles.

$$g(x)=\begin{cases} \$1.20 & 0\le x\le 1 \\ \$2.40 & 1<x\le 2 \\ \$3.60 & 2<x\le 3 \\ \$4.80 & 3<x\le 4 \\ \$6.00 & 4<x\le 5 \end{cases}$$

2. What is the slope of each interval? Explain your reasoning.

Slope is 0 No change = horizontal line

## 3. Graph g(x) for x < 5 miles.

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4. Describe the graph of the function as either increasing or decreasing.

Increasing

You have just graphed a *step function. A* ***step function*** is a piecewise function whose pieces are disjoint constant functions.

5. Why do you think this function is called a step function?

## Problem 3 Special Step Functions

The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**, $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

1. Evaluate each using the greatest integer function.

**a.** $\lfloor 2 \rfloor =$ 2

**b.** $\lfloor 0.17 \rfloor =$ 0

**c.** $\lfloor 2.34 \rfloor =$ 2

**d.** $\lfloor -1.2 \rfloor =$ -2

**e.** $\lfloor 2.99999 \rfloor =$ 2

**f.** $\lfloor -0.2 \rfloor =$ -1

**2.** Graph $G(x) = \lfloor x \rfloor$.