11-2 Inverses

Objective: I can find the inverse of a linear and cubic function.

Objective: I can verify inverses using composition.

Objective: I can explain the identity function and what it means for a function to be one-to-one.

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

Notation:

$$f^{-1}(x)$$

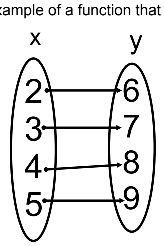
Represents the inverse of the function $f(x)$

One-to-One Definition:

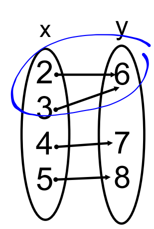
A function is one-to-one, if there is exactly one x for every y value (in addition to there being exactly one y for every x).

If the inverse of a function is a function, then the function is **one-to-one.**

Example of a function that is one-to-one.



Example of a function that is NOT one-to-one.



Find the inverse of each function. State whether the function is one-to-one.

a. $\{(5, 2), (4, 3), (3, 4), (2, 5)\}$

$\{(2,5), (3,4), (4,3), (5,2)\}$ one-to-one

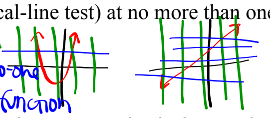
c. $\{(1, 2), (4, 3), (2, -1), (5, 3)\}$

$\{(2,1), (3,4), (-1,2), (3,5)\}$ NOT one-to-one

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

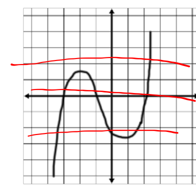
function
not one-to-one
Inverse: not function



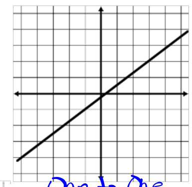
function
one-to-one
Inverse: function

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

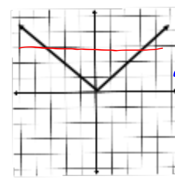
Determine whether the function is one-to-one.



not one-to-one



One to One



Not one to one

Inverse Functions

Function: $f(x) = 2x + 3$

UNDO

$x \xrightarrow{\text{Multiply by 2}} 2x \xrightarrow{\text{Add 3}} 2x+3$

Inverse Function: $f^{-1}(y) = \dfrac{y-3}{2}$

$\dfrac{y-3}{2} \xleftarrow{\text{Divide by 2}} y-3 \xleftarrow{\text{Subtract 3}} y$

$f(x) = x+2$

To find the inverse equation of a function

1. Change $f(x)$ to y. $y = x+2$
2. Interchange x and y $x = y+2$
   $-2 \quad -2$
3. Solve for y $x-2 = y$
4. Change new y to $f^{-1}(x)$ $f^{-1}(x) = x-2$

Finding Inverses

1. $f(x) = 3x + 4$

$y = 3x + 4$

$x = 3y + 4$

$-4 \qquad -4$

$\frac{x-4}{3} = \frac{3y}{3}$

$y = \frac{x-4}{3}$

$f^{-1}(x) = \frac{x-4}{3}$

2. $f(x) = \frac{x+1}{2}$

$y = \frac{x+1}{2}$

$2x = \left(\frac{y+1}{\cancel{2}}\right)\cancel{2}$

$2x = y + 1$

$-1 \qquad -1$

$2x - 1 = y$

$f^{-1}(x) = 2x - 1$

Find the inverse of each function.

1. $h(x) = 2x^3 + 3$

$y = 2x^3 + 3$

$x = 2y^3 + 3$

$-3 \qquad -3$

$\frac{x-3}{2} = \frac{2y^3}{2}$

$\sqrt[3]{\frac{x-3}{2}} = \sqrt[3]{y^3}$

$f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

2. $g(x) = \sqrt[3]{x} - 3$

You try

Find the inverse of the following functions

1. $f(x) = 12x - 1$

$y = 12x - 1$

$x = 12y - 1$

$\frac{x+1}{12} = \frac{12y}{12}$

$\frac{x+1}{12} = y$

$f^{-1}(x) = \frac{x+1}{12}$

2. $f(x) = x^3 - 6$

$x = y^3 - 6$

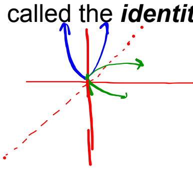
$+6 \qquad +6$

$\sqrt[3]{x+6} = \sqrt[3]{y^3}$

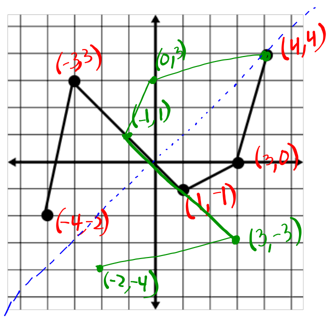
$f^{-1}(x) = \sqrt[3]{x+6}$

The graph of a function and its inverse is symmetrical with respect to the $y = x$ line.

This is called the ***identity function.***



Graph the inverse of the graph. (Use y=x to find inverse points)



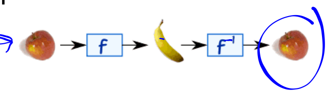
Inverses give you back the original value

$f(x)=x^2$
$g(x)=\sqrt{x}$

$f^{-1}(\ f(x)\ ) = x$

$f(\ f^{-1}(x)\ ) = x$

Examples

1. 

2. $f(x)=2x+3$

if x=4, then f(4) = 2(4)+3 = 11

$f^{-1}(y)=\dfrac{y-3}{2}$ $\quad f^{-1}(11)=\dfrac{11-3}{2}=\dfrac{8}{2}=4$

We can verify that two functions are inverses of each other by determining if the composition of the two functions are both equal to x.

$f \circ g = x \qquad g \circ f = x$

$f \circ f^{-1} = x \qquad f^{-1} \circ f = x$

Use composition to determine if the following functions are inverses of each other.

a) $f(x)=5x+1$

$g(x)=\dfrac{x-1}{5}$

$f(g(x))=\cancel{5}\left(\dfrac{x-1}{\cancel{5}}\right)+1=x-1+1=x$

$g(f(x))=\dfrac{5x+1-1}{5}=\dfrac{\cancel{5}x}{\cancel{5}}=x$

b) $f(x)=\dfrac{x-1}{4}$

$g(x)=4x+1$

$f(g(x))=\dfrac{4x+\cancel{1}-\cancel{1}}{\cancel{4}}=x$

$g(f(x))=\cancel{4}\left(\dfrac{x-1}{\cancel{4}}\right)+1$

$x-1+1=x$

Use composition to determine if the following functions are inverses of each other.

a) $f(x) = x^3 - 6$

$g(x) = \sqrt[3]{x+6}$

$f(g(x)) = \left(\sqrt[3]{x+6}\right)^3 - 6$

$= x + \cancel{6} - \cancel{6}$

$= x$

$g(f(x)) = \sqrt[3]{x^3 - 6 + 6}$

$\sqrt[3]{x^3}$

$g(f(x)) = x$

b) $f(x) = 3x^3 + 9$

$g(x) = \dfrac{\sqrt[3]{3x-27}}{3}$

(If it comes up....)

$f(x) = x^2$