10.5 Factoring Quadratics

To **factor** a quadratic expression means to write it as the **product** of two **binomial** expressions.

To factor, we can FOIL backwards. For a quadratic of the form, 
\[ x^2 + bx + c \]
we need to find numbers \( p \) and \( q \) such that:
\[ p + q = b \text{ and } pq = c \]
To factor a quadratic, look at $b$ and $c$.

1. Find two numbers whose **product** is $c$ and whose **sum** is $b$.

2. Look at the signs. If the second sign is **positive**, then the signs in factored form are **positive**. If the second sign is **negative**, then the signs in factored form are **negative**.

3. If the signs in factored form are the same, look at the first sign. This sign tells us what sign to use in both factors. If the signs in factored form are different, look at the first sign. This sign tells us to make the **first** number that sign.

4. Check the factoring using FOIL.
Factor: $x^2 + 7x + 10$

\[
\begin{array}{c}
10 \\
\nearrow \\
5 + 2 \\
\nearrow \\
-5 - 2 \\
\nearrow \\
-1 - 10
\end{array}
\]

\[
\begin{array}{c}
10 \\
\nearrow \\
1 + 10 \\
\nearrow \\
10
\end{array}
\]

\[
(x+5)(x+2)
\]

\[
\begin{align*}
x^2 + 2x + 6x + 10 \\
x^2 + 7x + 10
\end{align*}
\]
Factor: \[ x^2 + 2x - 15 \]

\[
\begin{bmatrix}
-15 & -15 \\
-3 + 5 & -5 + 3 \\
-15 & -15 \\
1 & 15 & 15
\end{bmatrix}
\]

\[ (x-3)(x+5) \]

Check: \[ x^2 + 5x - 3x - 15 \]
\[ x^2 + 2x - 15 \]
Factor: \( x^2 + 7x + 12 \)

\[
\begin{array}{c}
12 \quad 12 \\
\times \\
2 \quad 6 \\
3 + 4 = 7
\end{array}
\]

\(2 \times \frac{1}{2} = 1 \)

\[ (x+3)(x+4) \]
Factor: \(x^2 + 11x - 26\)

\[
\begin{array}{c c c}
-26 & -26
\end{array}
\]

\[
\begin{array}{c c c}
-1 & 26 & -26
\end{array}
\]

\[
\begin{array}{c c c}
-26 & -26
\end{array}
\]

\[
\begin{array}{c c c}
13 & -2
\end{array}
\]

\[
(x + 13)(x - 2)
\]
3) \( t^2 - 10t + 21 \)

\[
\begin{array}{c}
21 \\
\vee \\
-7, -3, -1, -21
\end{array}
\]

\( (x-7)(x-3) \)

IF the last # is +

and the middle # is -

Then both factors are -