10-4

Solving Quadratic Inequalities

Objectives:

1. I can solve a quadratic inequality.

2. I can graph the answer to a quadratic inequality.

3. I can state my answer in set and interval notation.

Replace the ? with <, >, or = to make the statement true.

1. 3 ? 6
   $<$

2. -3 ? -6
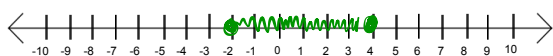 $>$

3. $\frac{1}{2}$? .5
   $=$

4. $\frac{2}{3}$? $\frac{3}{5}$
   $>$

Write each inequality using interval notation then graph.

Set — such That

$\{x \mid -2 \le x \le 4\}$ $[-2, 4]$

Tells you the variable   restrictions on variable

$\{x \mid 1 < x \le 5\}$ $(1, 5]$

Write each inequality using interval notation then graph.

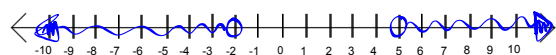$\{x \mid x < 2\}$ $(-\infty, 2)$
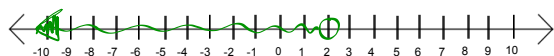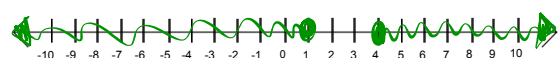

$\{x \mid x \ge 3\}$ $[3, \infty)$

Write each inequality using interval notation then graph.

$\{x \mid x < -2 \cup x > 5\}$ $(-\infty, -2) \cup (5, \infty)$
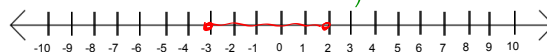
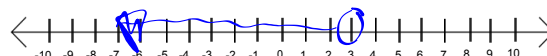$\{x \mid x \le 1 \cup x \ge 4\}$ $(-\infty, 1] \cup [4, \infty)$

You try

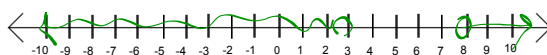$\{x \mid -3 \le x \le 2\}$ $[-3, 2]$
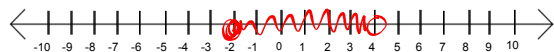
$\{x \mid x < 3\}$ $(-\infty, 3)$

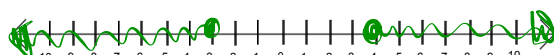$\{x \mid x < 3 \cup x > 8\}$ $(-\infty, 3) \cup (8, \infty)$

Write each interval using inequality notation involving x, then graph.

$[-2, 4)$ $\{x \mid -2 \le x < 4\}$

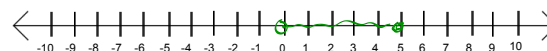$(-\infty, -3] \cup [4, \infty)$ $\{x \mid x \le -3 \cup x \ge 4\}$

$(-\infty, 1)$ $\{x \mid x < 1\}$
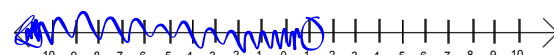
You try

$(0, 5]$ $\{x \mid 0 < x \le 5\}$
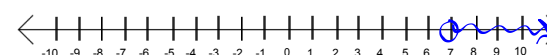
$(-\infty, -6) \cup (3, \infty)$ $\{x \mid x < -6 \cup x > 3\}$

$(7, \infty)$ $\{x \mid x > 7\}$

Solve the following using the algebraic method.

$$x^2 - 4x - 5 \geq 0$$

$-5 \quad 1$

$$(x-5)(x+1) \geq 0$$

$$x = 5, -1$$

$+ \quad - \quad +$

$-1 \quad 5$
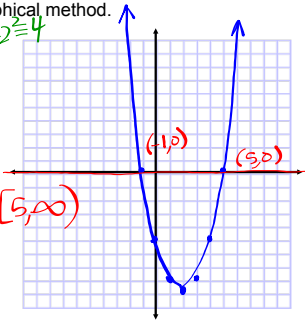
$$(-\infty, -1] \cup [5, \infty)$$

Solve the following using the graphical method.

$$x^2 - 4x - 5 \geq 0 \quad \frac{-4}{2} = -2 \quad -2^2 = 4$$

$$(x^2 - 4x + 4) - 5 - 4$$

$$(x-2)^2 - 9 \geq 0$$

$$(-\infty, -1] \cup [5, \infty)$$

$(-1, 0)$ $(5, 0)$

You Try
Solve algebraically then check your answer by graphing

$$x^2 + 3x - 10 \geq 0$$

$-2 \quad 5$

$$(x-2)(x+5) \geq 0$$

$$x = 2, -5$$

$+ \quad - \quad +$

$-5 \quad 2$

$$(-\infty, -5] \cup [2, \infty)$$

You Try
Solve algebraically then check your answer by graphing

$$-x^2 + 5x - 6 < 0$$

$$-(x^2 - 5x + 6) < 0$$

$-2 \quad -3$

$$-(x-2)(x-3) < 0$$

$- \quad + \quad -$

$2 \quad 3$

$$(-\infty, 2) \cup (3, \infty)$$

Panel 1 (top left):

Solve algebraically then check your answer by graphing

$$2x^2 > 4x - 1$$

$-4x+1 \quad -4x+1$

$2x^2 - 4x + 1 > 0$

$x = \dfrac{4 \pm \sqrt{16 - 4(2)(1)}}{4}$

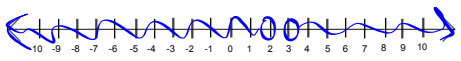$x = \dfrac{4 \pm \sqrt{8}}{4}$

$x = \dfrac{4 \pm 2\sqrt{2}}{4}$

$x = 2 \pm \sqrt{2}$

$\dfrac{2 - \sqrt{2}}{2} \qquad \dfrac{2 + \sqrt{2}}{2}$

$+ \quad | \quad - \quad | \quad +$

$\left(-\infty, \dfrac{2-\sqrt{2}}{2}\right) \cup \left(\dfrac{2+\sqrt{2}}{2}, \infty\right)$

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Panel 2 (top right):

Solve algebraically then check your answer by graphing

$$3x^2 < -x + 5$$

$+x - 5 \quad +x - 5$

$3x^2 + x - 5 \leq 0$

$x = \dfrac{-1 \pm \sqrt{1 + 4(3)(5)}}{6}$

$x = \dfrac{-1 \pm \sqrt{61}}{6}$

$\dfrac{-1 - \sqrt{61}}{6} \qquad \dfrac{-1 + \sqrt{61}}{6}$

$+ \quad - \quad +$

$\left(\dfrac{-1-\sqrt{61}}{6}, \dfrac{-1+\sqrt{61}}{6}\right)$

$-1.5 \qquad 1.2$

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Panel 3 (bottom left):

Solve algebraically then check your answer by graphing

$$y^2 + 3y + 5 \geq 0$$

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Panel 4 (bottom right):

Solve algebraically then check your answer by graphing

$$y^2 + 3y + 5 \geq 0$$

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Solve algebraically then check your answer by graphing

$$x^2 + 8x + 16 > 0$$