

Def: 2 relations  $f$  &  $g$  are inverses iff both of their compositions are the identity function ( $y=x$ )

this means  $f(g(x))=x$  &  $g(f(x))=x$

- it does not mean they equal the same thing -  
they must equal  $x$ !!

**Notation:**  $f^{-1}(x)$ : inverse of  $f(x)$

(not an exponent,  $f$  is the name of a function not a variable!!)

Use the definition of inverses to show that  $f(x) = 6 - 2x$  &

$g(x) = \frac{6-x}{2}$  are inverses:

$$f(g(x)) = f\left(\frac{6-x}{2}\right) = 6 - 2\left(\frac{6-x}{2}\right) = 6 - 6 + x = x$$
$$g(f(x)) = g(6-2x) = \frac{6-(6-2x)}{2} = \frac{0+2x}{2} = x$$

## Inverses - graphically (card)

Inverse relations are reflections of each other over the line  $y = x$  (identity function)

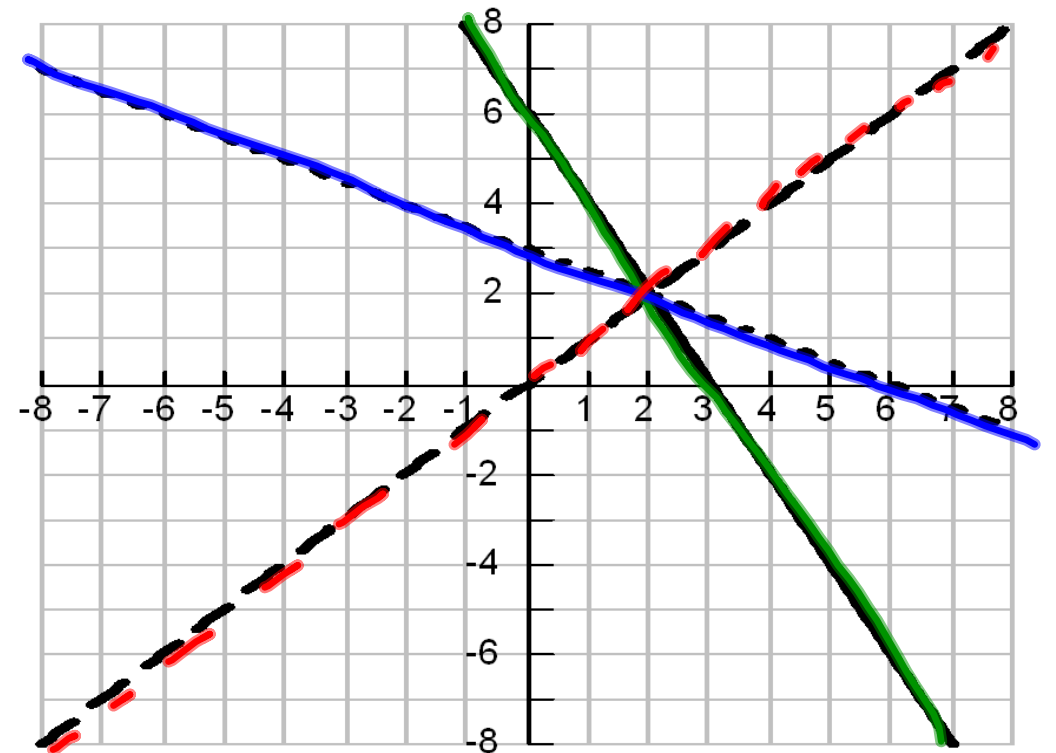
mirror images over  $y = x$

so if  $g(x)$  and  $f(x)$  are inverses then every point  $(a,b)$  if  $f(x)$  will be reflected onto its mirror image  $(b,a)$  in  $g(x)$  and vice versa

★ Property of inverse relations: Suppose  $f$  &  $f^{-1}$  are inverse relations, then  $f(a)=b$  iff  $f^{-1}(b)=a$

back

Graph to show  $f(x) = 6 - 2x$  and  $g(x) = \frac{6 - x}{2}$   
are inverses

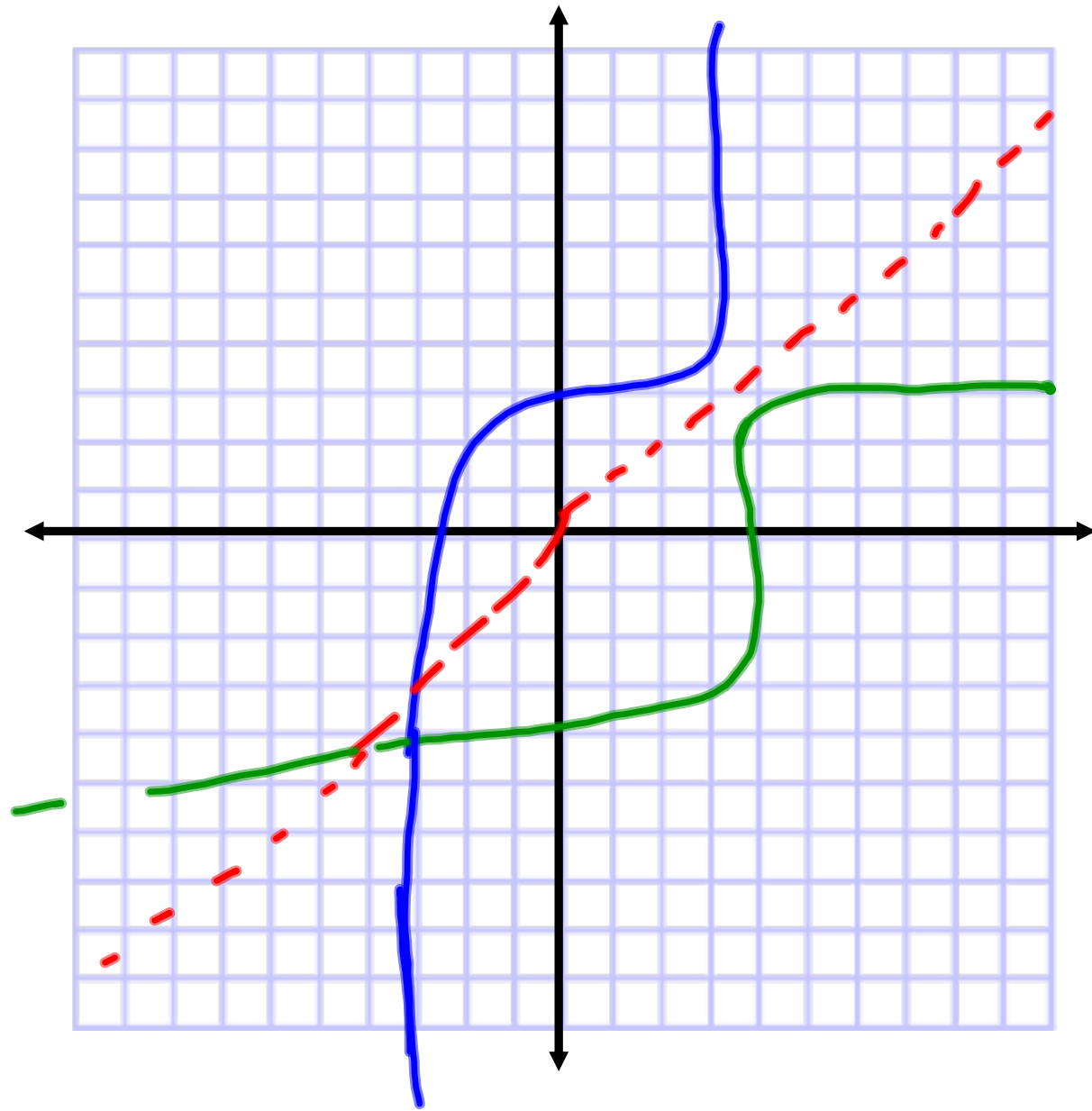


Fold at  $y = x$ , graphs will  
lie on top of each other if  
they are inverses or  
Look at pts:

f	(1,4)	(3,0)	(4,-2)
g	(4,1)	(0,3)	(-2,4)

Red arrows indicate that the points for f and g are reflections across the line y=x. For example, (1,4) on f reflects to (4,1) on g, (3,0) reflects to (0,3), and (4,-2) reflects to (-2,4).

## Finding inverses graphically



1. Draw in  $y=x$
2. reflect  $f$  over  $y=x$

# Finding an Inverse Algebraically (card)

## Steps:

1. replace  $f(x)$  or relation name w/  $y$  if not in that form
2. switch the  $x$  &  $y$  in the eq. (just  $x$  &  $y$  not signs, coefficients, or exponents)
3. Solve for  $y$ .
4. replace  $y$  with relation name<sup>-1</sup> ( $f^{-1}$  or  $g^{-1}$ )

back

find the inverse

$$f(x) = x^2 + 1$$

$$y = x^2 + 1$$

$$x = y^2 + 1$$

$$\sqrt{x-1} = \sqrt{y^2}$$

$$\pm\sqrt{x-1} = y$$

$$f^{-1}(x) = \pm\sqrt{x-1}$$

$$g(x) = \frac{x+1}{2x+3}$$

$$y = \frac{x+1}{2x+3}$$

$$(2y+3)x = \frac{y+1}{2y+3} (2y+3)$$

$$2yx + 3x = y + 1$$

$$2yx - y + 3x = 1$$

$$2yx - y = 1 - 3x$$

$$\frac{y(2x-1)}{2x-1} = \frac{1-3x}{2x-1}$$

$$y = \frac{1-3x}{2x-1}$$

$$g^{-1}(x) = \frac{1-3x}{2x-1}$$

# Is the inverse a function????

## How can I tell?

### One-to-One (card)

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- a function whose inverse is also a function

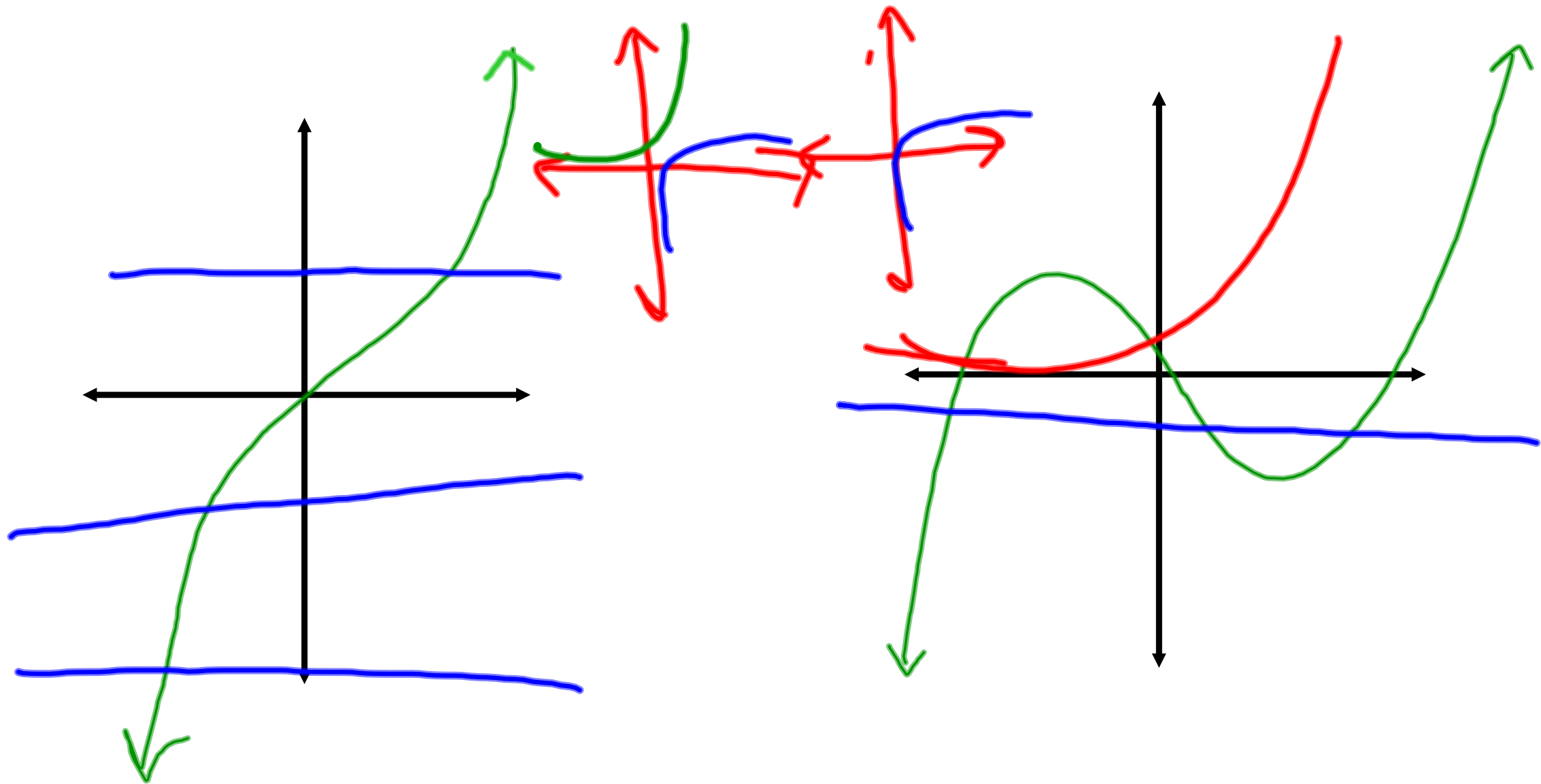
a function such that each  $x$  is paired with only one  $y$  and each  $y$  is paired with only one  $x$

must pass the horizontal line test to be one to one



(back of one to one card)

Determine if each function is one to one



$$f(x) = \sqrt{2x - 3}$$

$$2x - 3 \geq 0 \quad x \geq 3/2$$

$$2x \geq 3$$

Find the inverse and the domain of  $f^{-1}$  including any inherited restrictions

$$D: [3/2, \infty)$$

$$R: [0, \infty)$$

$$y = \sqrt{2x - 3}$$

$$x \geq 3/2$$

$$y \geq 0$$

$$x \geq 0$$

$$\hat{x}^2 = \sqrt{2y - 3}^2$$

$$x^2 = 2y - 3$$

$$\frac{x^2 + 3}{2} = 2y$$

$$\frac{x^2 + 3}{2} = y$$

$$f^{-1}(x) = \frac{x^2 + 3}{2}$$

$$D: [0, \infty) \quad R: [3/2, \infty)$$