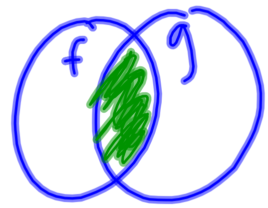
1.4 Building Functions from Functions

We can find new functions by building them from other functions using operations we are already familiar with.

Operations on Functions

Two functions f and g have intersecting domains, then for all values of x in that intersection...



Sum: $(f+g)(x)=f(x)+g(x)$

Difference: $(f-g)(x)=f(x)-g(x)$

Product: $(fg)(x)=f(x)g(x)$

Quotient: $(\frac{f}{g})(x)=\frac{f(x)}{g(x)}, g(x)\neq 0$

REMEMBER: The domain of the new function is all the x-values that are in BOTH the domain of f and the domain of g. (take special notice of zeros in the denominator when dividing)

$f(x) = x^2$ $g(x) = \sqrt{x+1}$

Domain: $(-\infty, \infty)$ Domain: $[-1, \infty)$

Which values are in both domains? $[-1, \infty)$

$(f+g)(x) = f(x) + g(x) = x^2 + \sqrt{x+1}$ D: $[-1, \infty)$

$(f-g)(x) = f(x) - g(x)$ $x^2 - \sqrt{x+1}$ D: $[-1, \infty)$

$(fg)(x) = f(x)g(x)$ $(x^2)(\sqrt{x+1})$ D: $[-1, \infty)$

$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ $\frac{x^2}{\sqrt{x+1}}$ D: $(-1, \infty)$

$(gg)(x) = g(x)g(x)$ $x+1$ D: $[-1, \infty)$

Composition of Functions - defined

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suppose f & g are functions such that the range of g is a subset of the domain of f, then the composite function that we write $f \circ g$ can be described by the following equation:

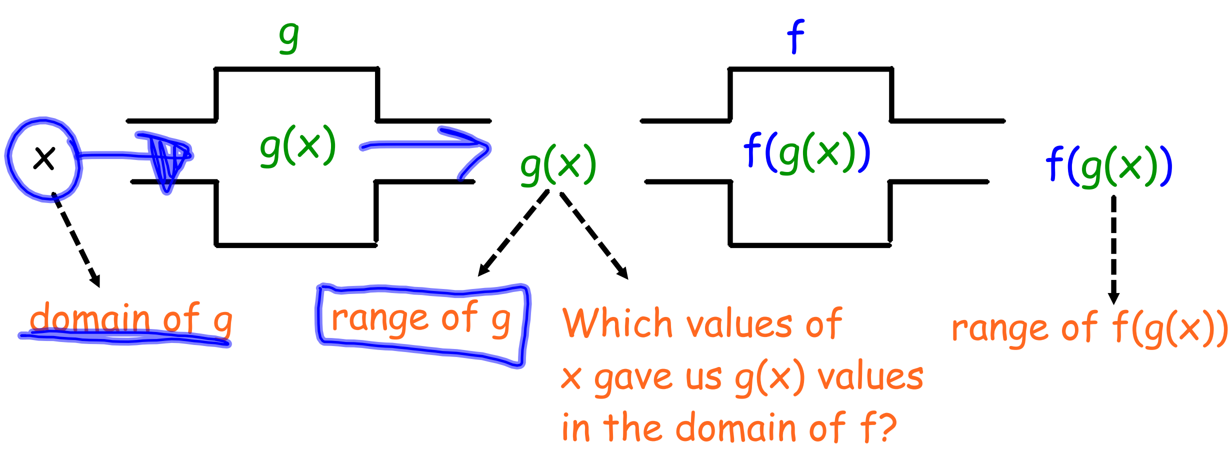
$$(f \circ g)(x) = f(g(x))$$

read "f of g"

work from the inside out. Compositions can be done with discrete and continuous functions.

Domain of Compositions

The domain of $(f \circ g)(x)$
consists of all x-values of g that map to g(x)-values in the domain of f



Discrete functions:

Given:
f={(1,3) (2,7) (3, -2)}
g={(7,11) (3,-6) (-2,-3) (5, 1)}

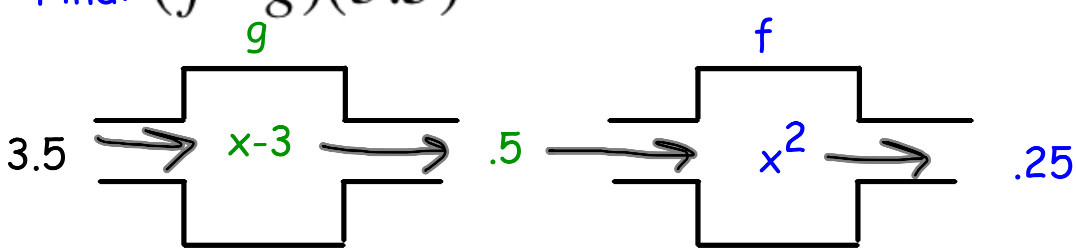
1. Find: $g \circ f(x) = g(f(x))$

2. Find: $f \circ g(x) = f(g(x))$

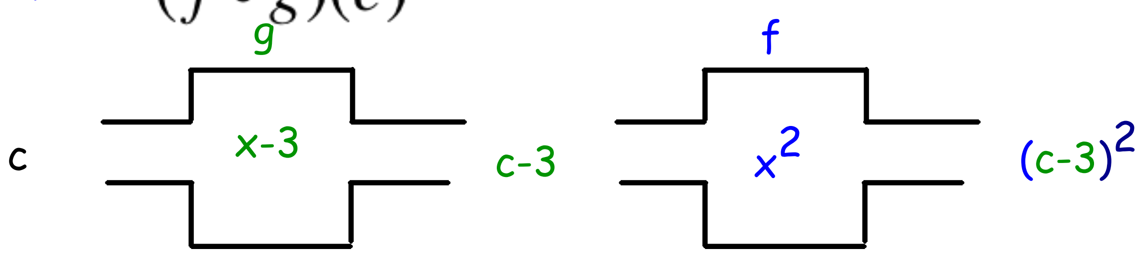
Continuous Functions:

$$f(x) = x^2 \qquad g(x) = x - 3$$

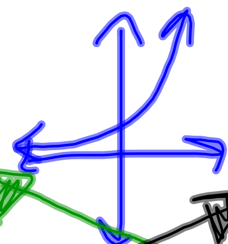
Find: $(f \circ g)(3.5)$



Find: $(f \circ g)(c)$



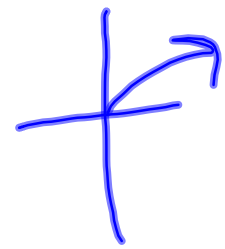
$f(x) = e^x$



D: $(-\infty, \infty)$

R: $(0, \infty)$

$g(x) = \sqrt{x}$



D: $[0, \infty)$

R: $[0, \infty)$

$(f \circ g)(x) = f(g(x)) = e^{\sqrt{x}}$

D: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = \sqrt{e^x}$

D: $(-\infty, \infty)$

Finding the domain of a composition

$f(x) = x^2 - 1$

D: $(-\infty, \infty)$

R: $[-1, \infty)$

$g(x) = \sqrt{x}$

D: $[0, \infty)$

R: $[0, \infty)$

$(f \circ g)(x)$ $f(g(x)) = (\sqrt{x})^2 - 1 = x - 1$

D: $[0, \infty)$

$(g \circ f)(x)$ $g(f(x)) = \sqrt{x^2 - 1}$

D: $(-\infty, -1] \cup [1, \infty)$

1. What is the domain of the first function?
2. Find the domain restrictions of the new function
3. Put them together

$f(x) = x^2 - 4$ $\qquad$ $g(x) = \sqrt{x - 3}$

Find f(g(x)), g(f(x)) and the domain of the composition.

Decomposition

given h(x) find functions f and g such that h(x)=f(g(x))

$$h(x) = (x+1)^2 - 3(x+1) + 4$$

$$h(x) = \sqrt{x^3 + 1}$$

$$f(g(x)) = \left(x^2 + 3\right)^2$$

given the composition - find f(x) and g(x)

$$h(x) = \sqrt{x^3 + 1}$$

given h(x) find f(x) and g(x) so that h(x)=f(g(x))

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