

# Characteristics of a Function:

Domain -  $x$ -value

Range -  $y$ -values

Increasing

Decreasing

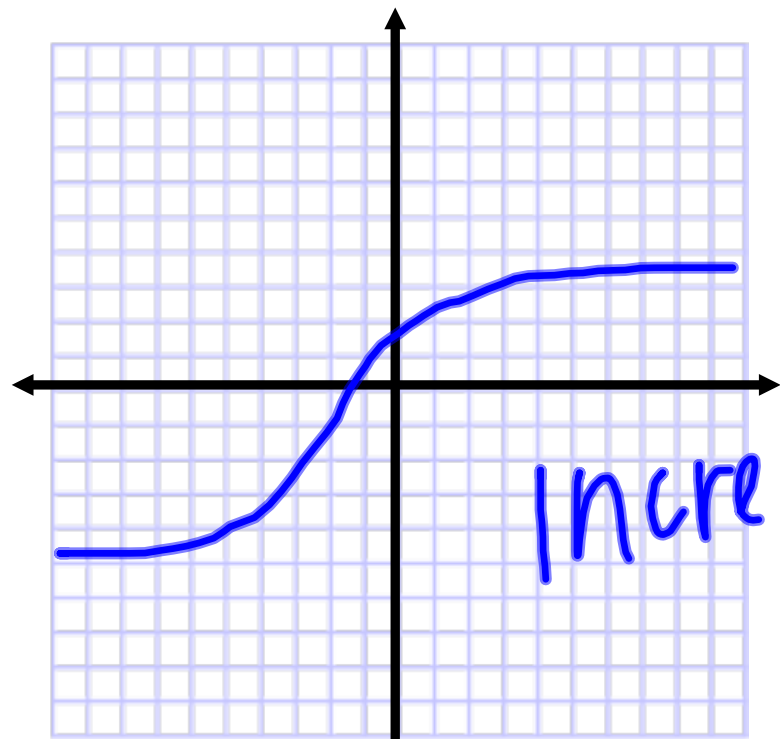
Extrema - max. and min.

Compare degrees  $\rightarrow$  H.A. V.A.  $\checkmark$  0 indennominator

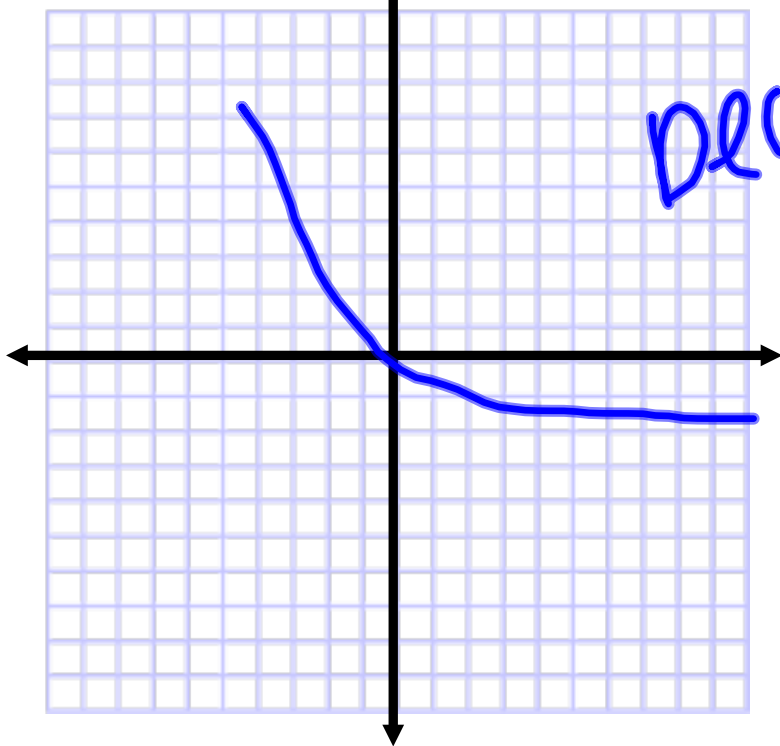
Continuity- asymptotes, holes, jumps

Symmetry - odd, even, neither

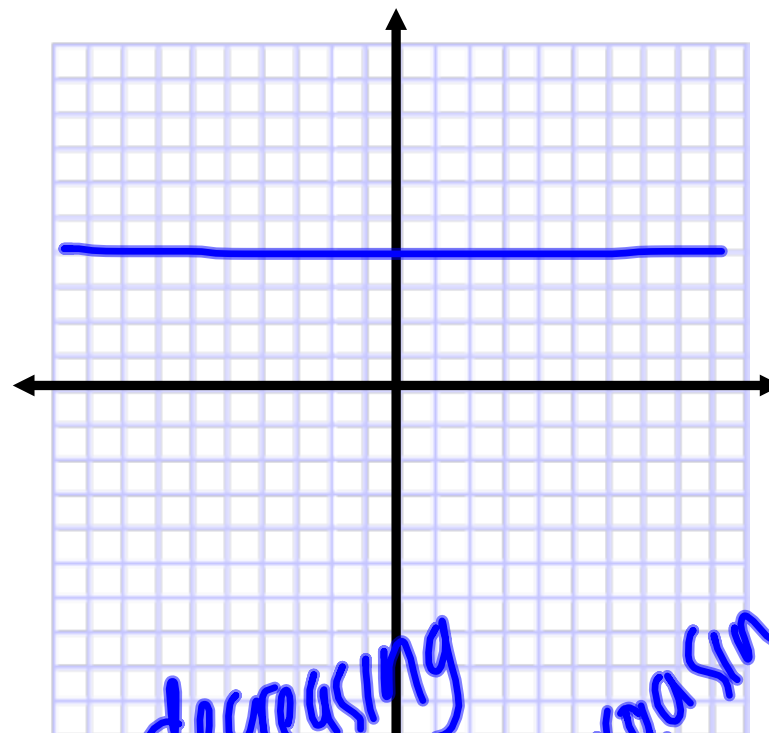
Removable



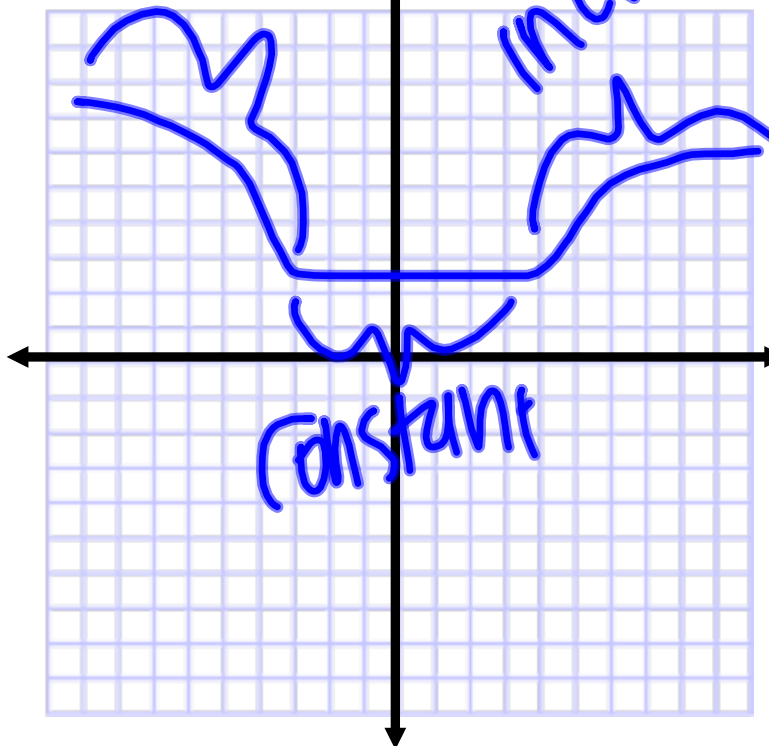
Increasing



Decreasing



constant



decreasing

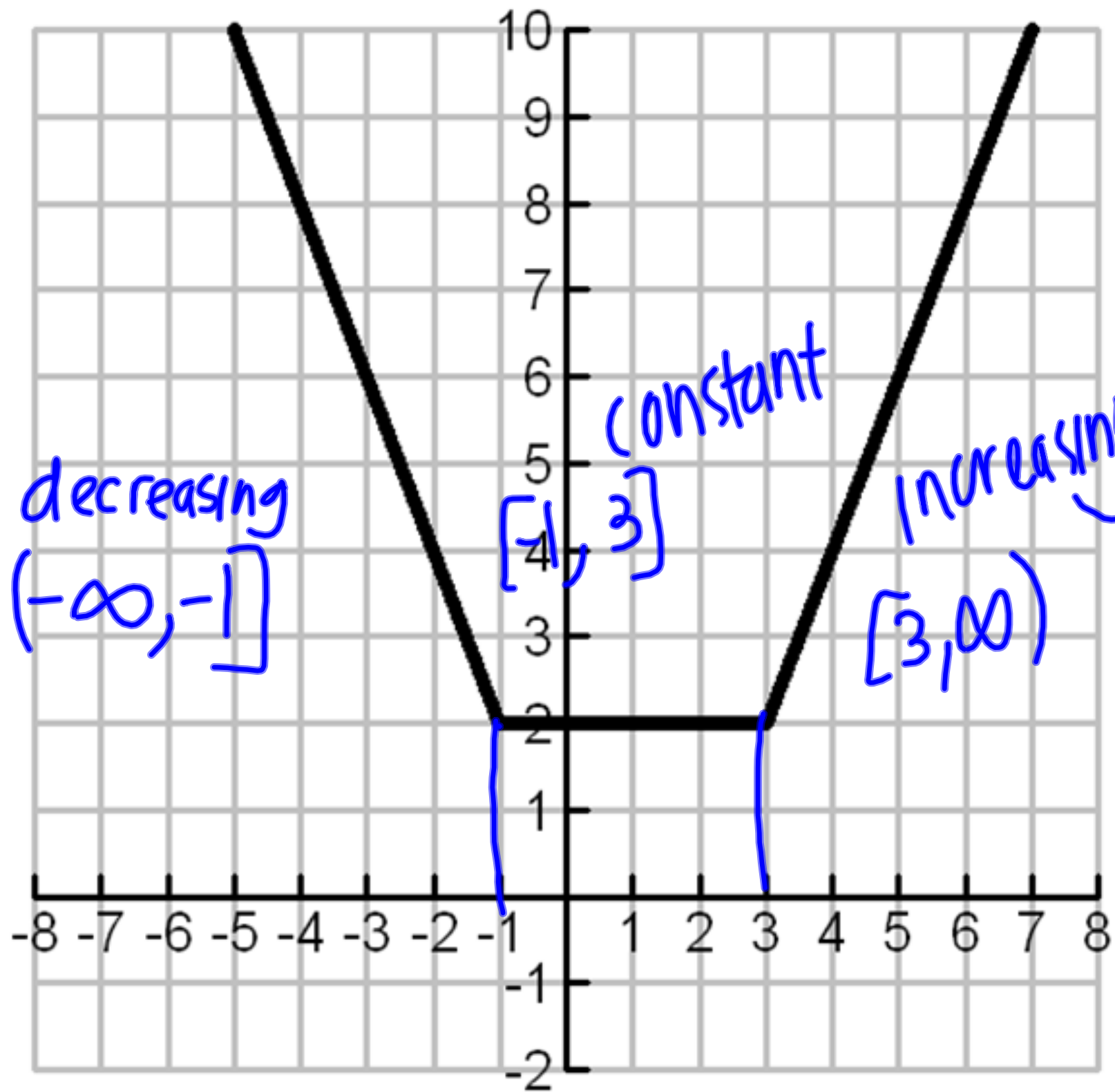
increasing

constant

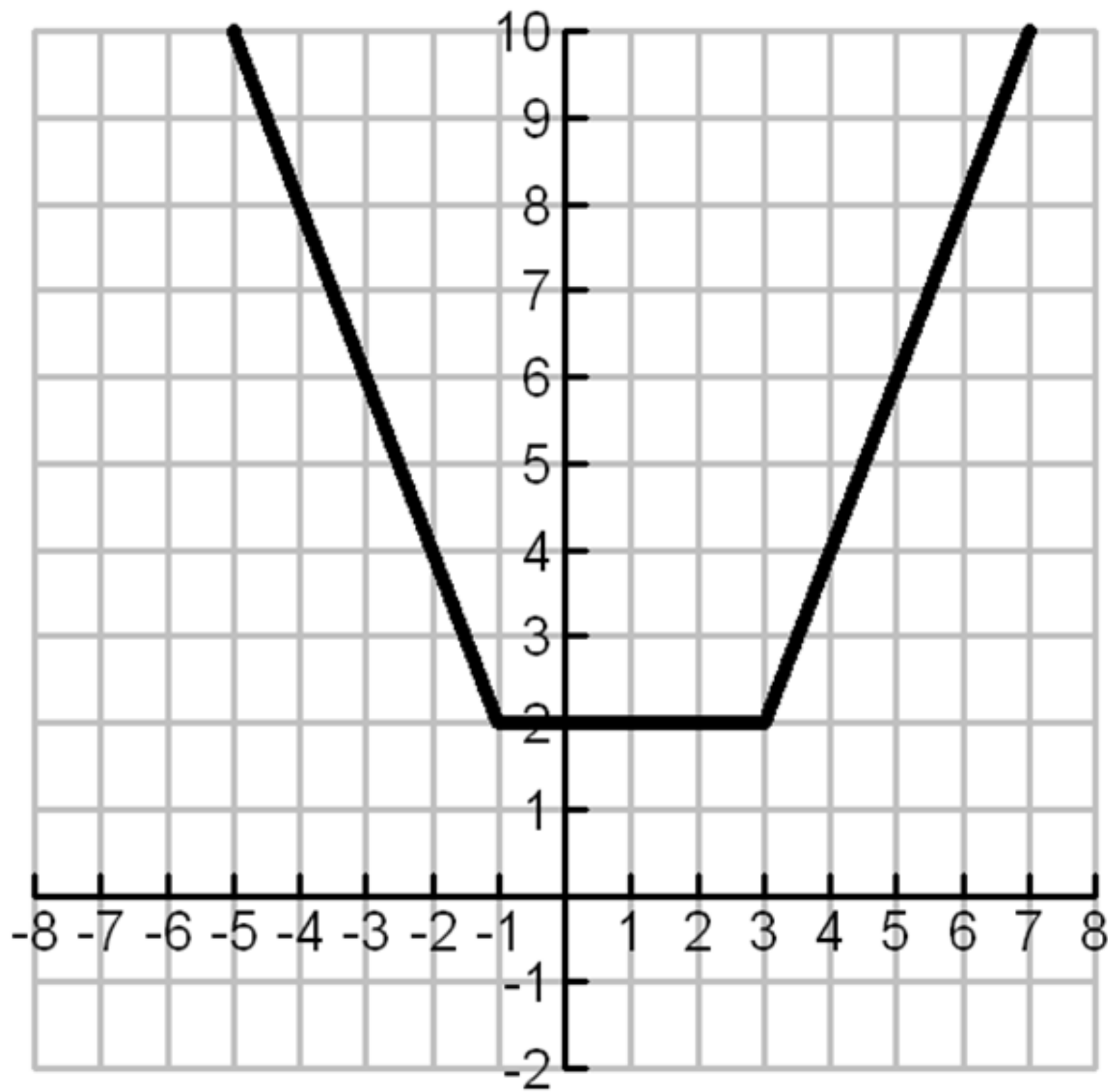
# Increasing, Decreasing and Constant

- as you move from left to right the y-values  
increase
- as you move from left to right the y-values  
decrease
- as you move from left to right the y-values do not  
change

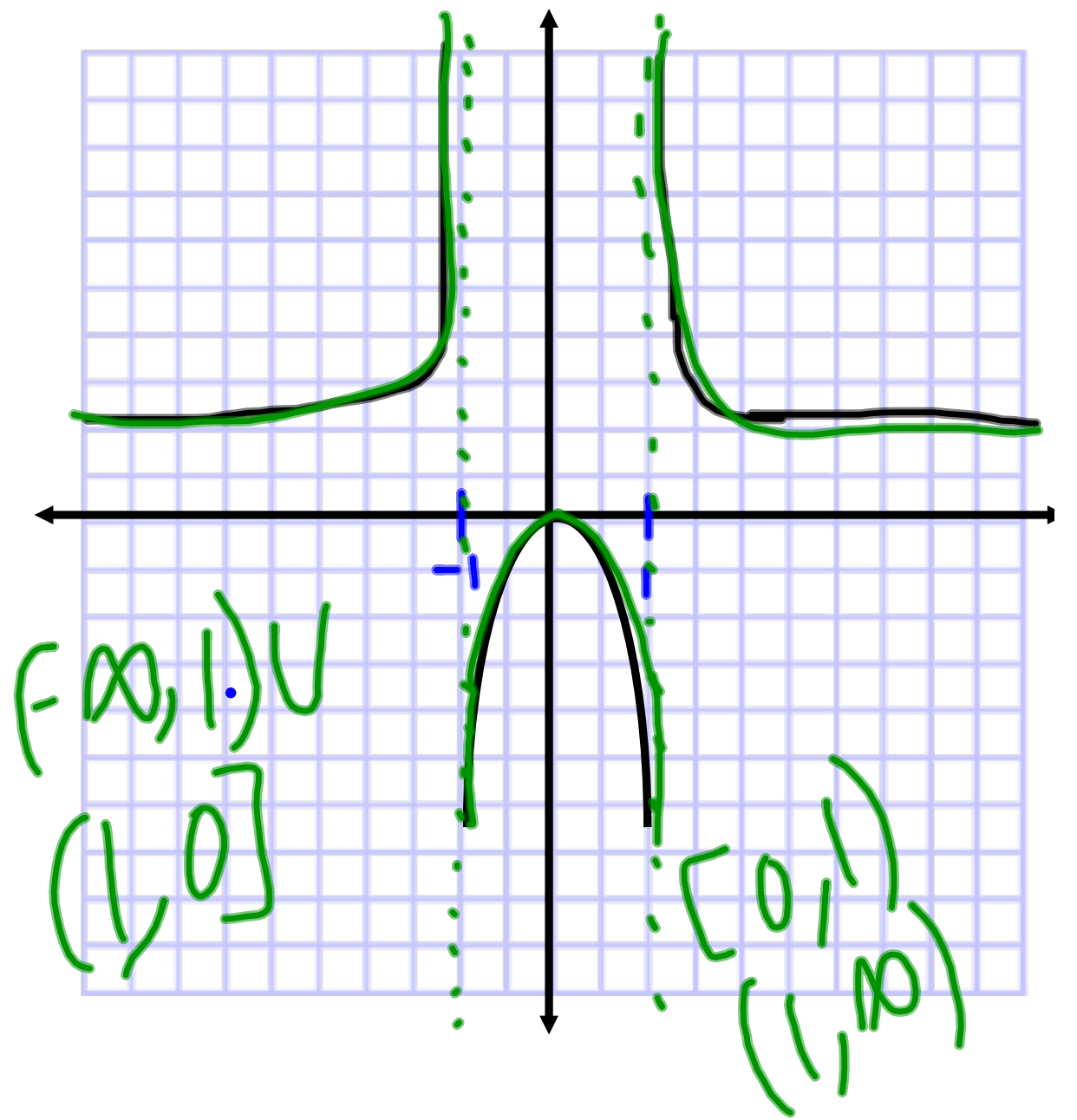
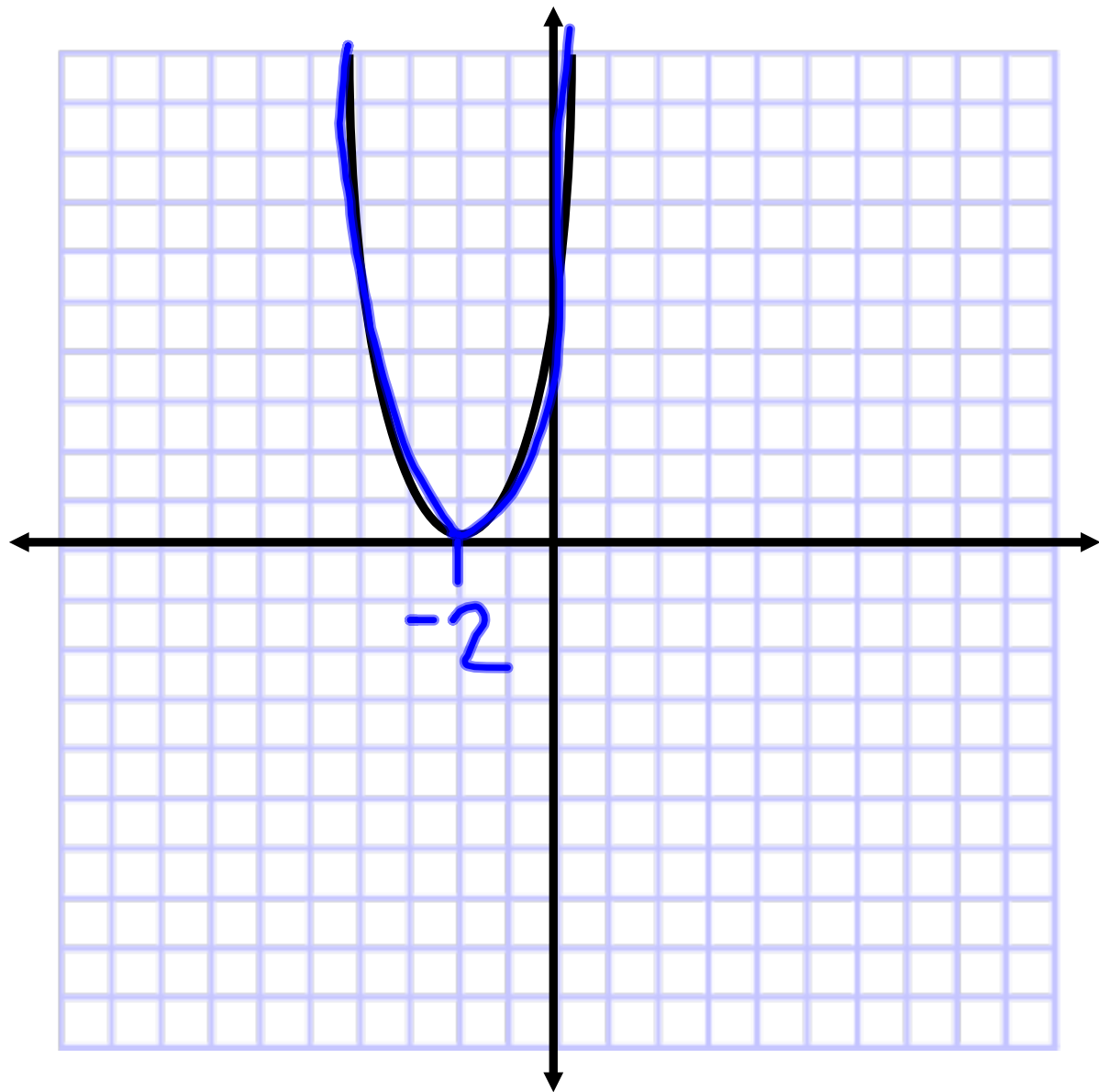
this behavior is reported using interval notation for the x-values where the graph has a certain behavior



x	$y_1(x)$ $\text{abs}(x - 2)$
-7	14
-6	12
-5	10
-4	8
-3	6
-2	4
-1	2
0	2
1	2
2	2
3	2
4	4
5	6
6	8
7	10
8	12
9	14
10	16
11	18
12	20
13	22
14	24
15	26
16	28



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-6	12
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-4	8
-3	6
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5	6
6	8
7	10
8	12
9	14
10	16
11	18
12	20
13	22
14	24
15	26
16	28



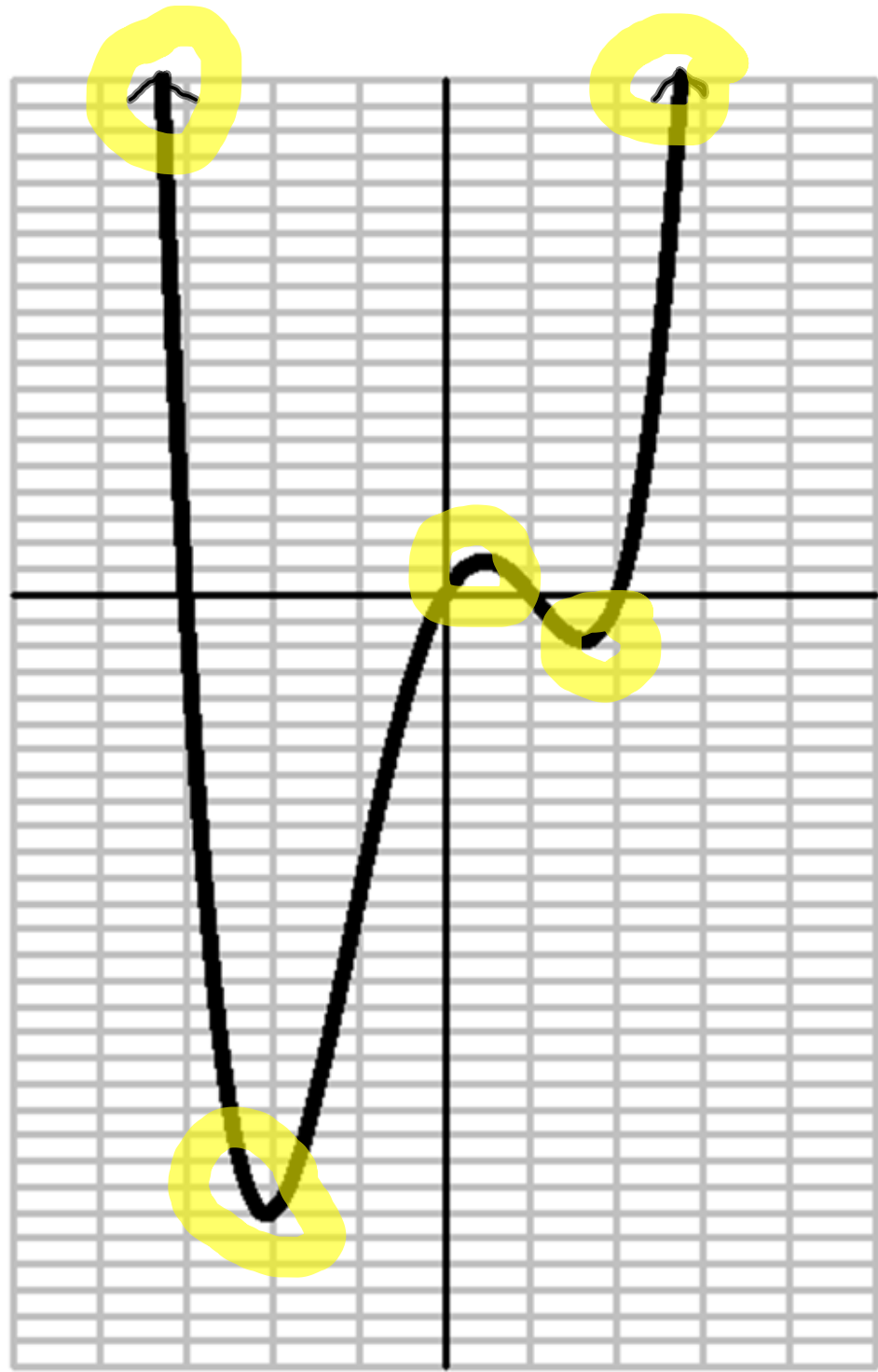
# Extrema

## maximums

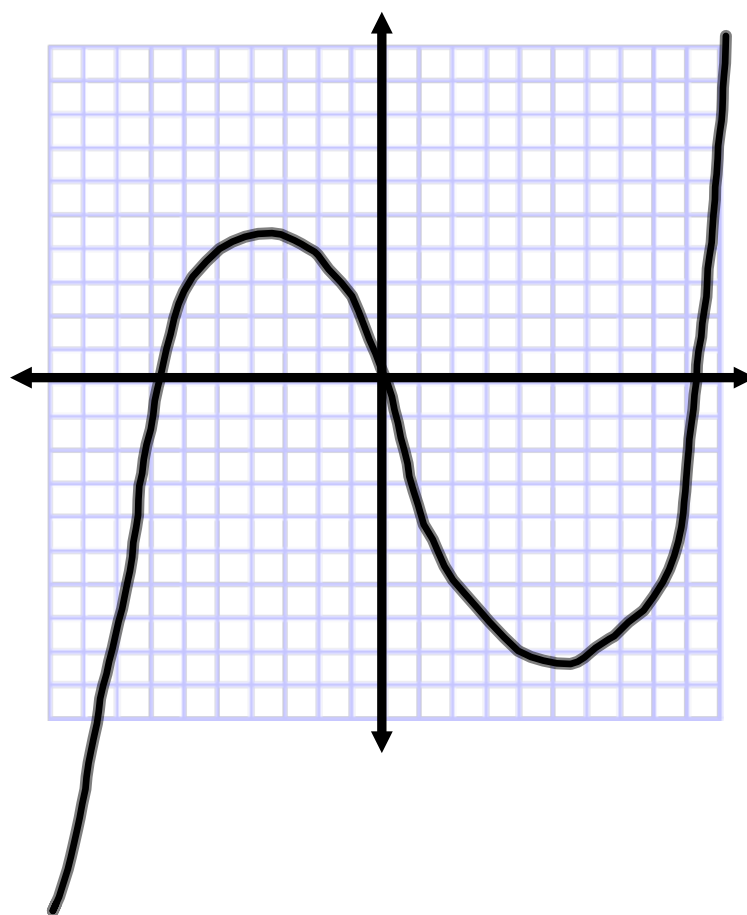
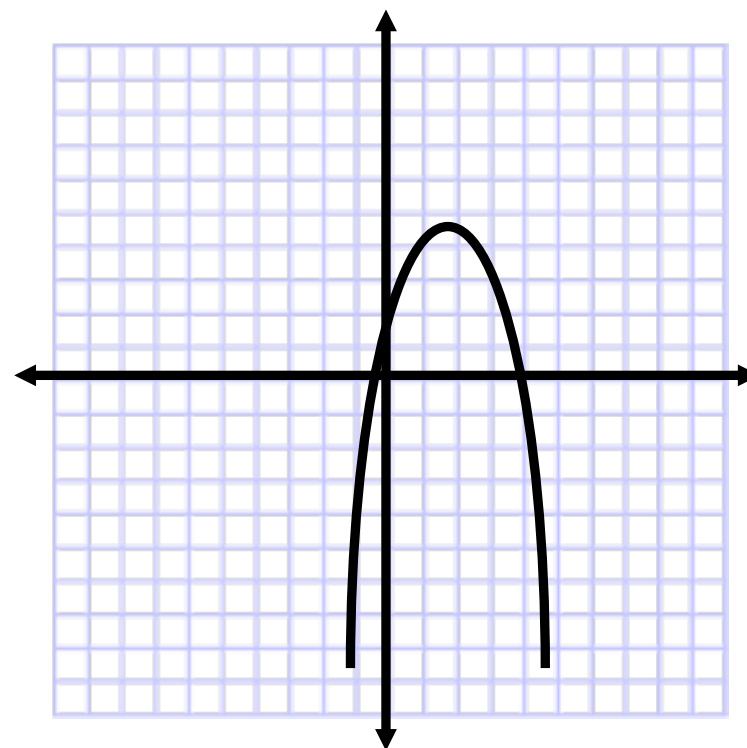
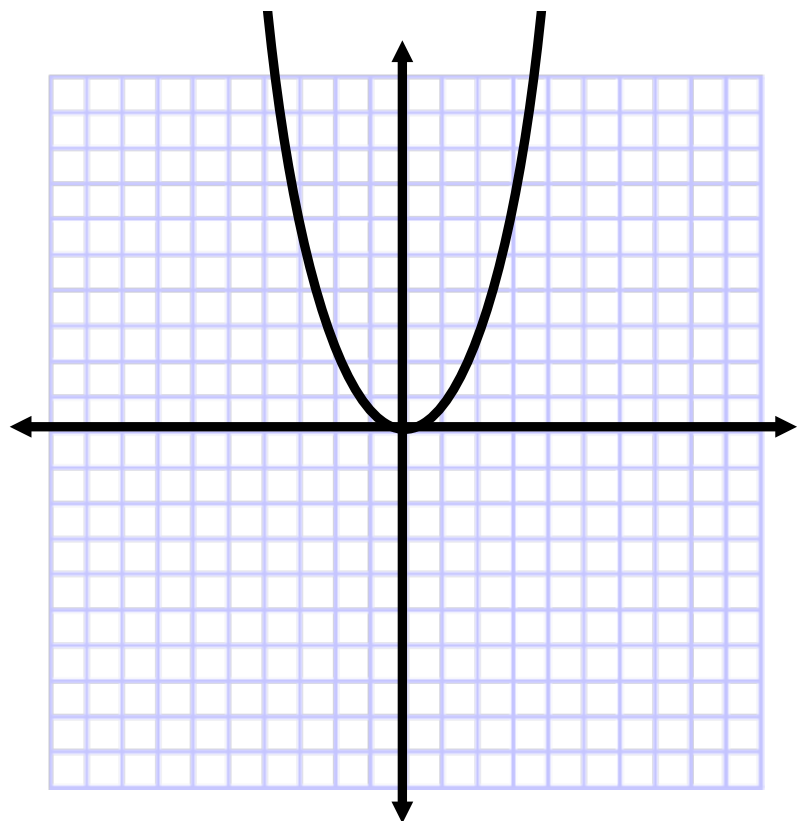
- relative (local)
- absolute (upper bound)

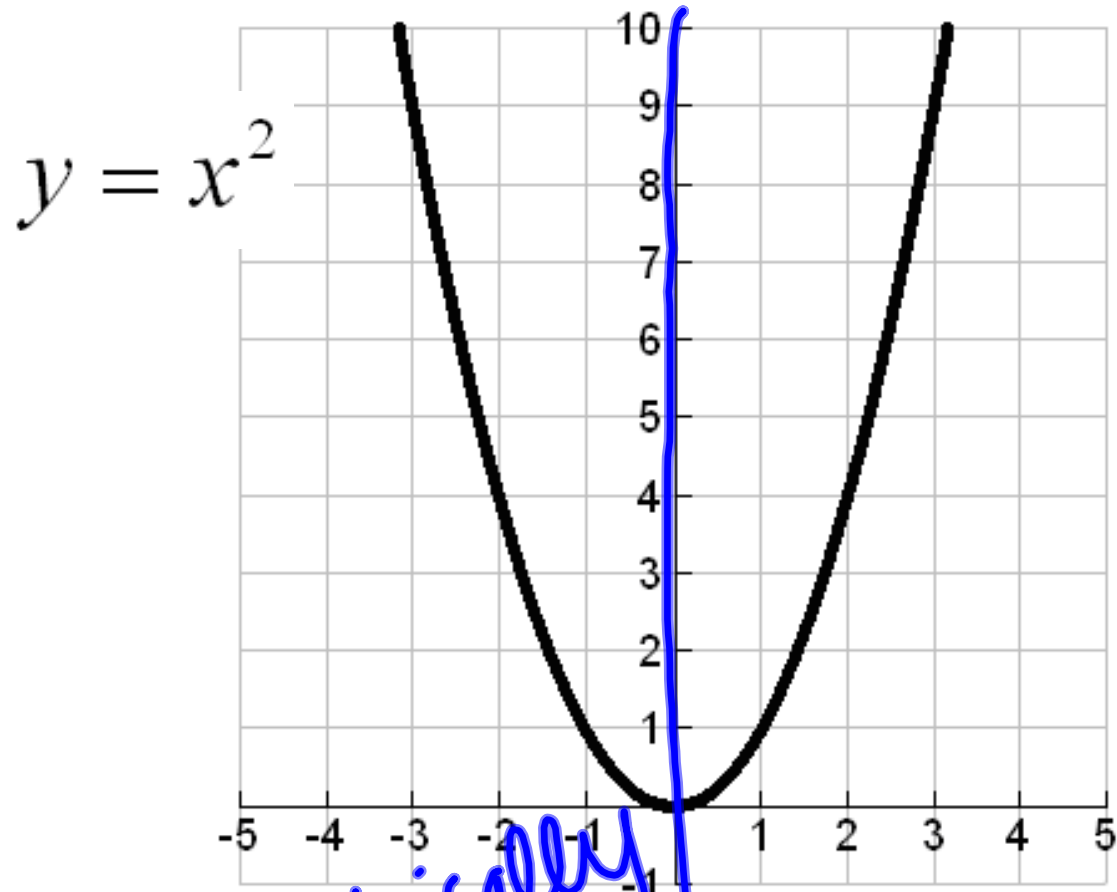
## minimums

- relative (local)
- absolute (lower bound)









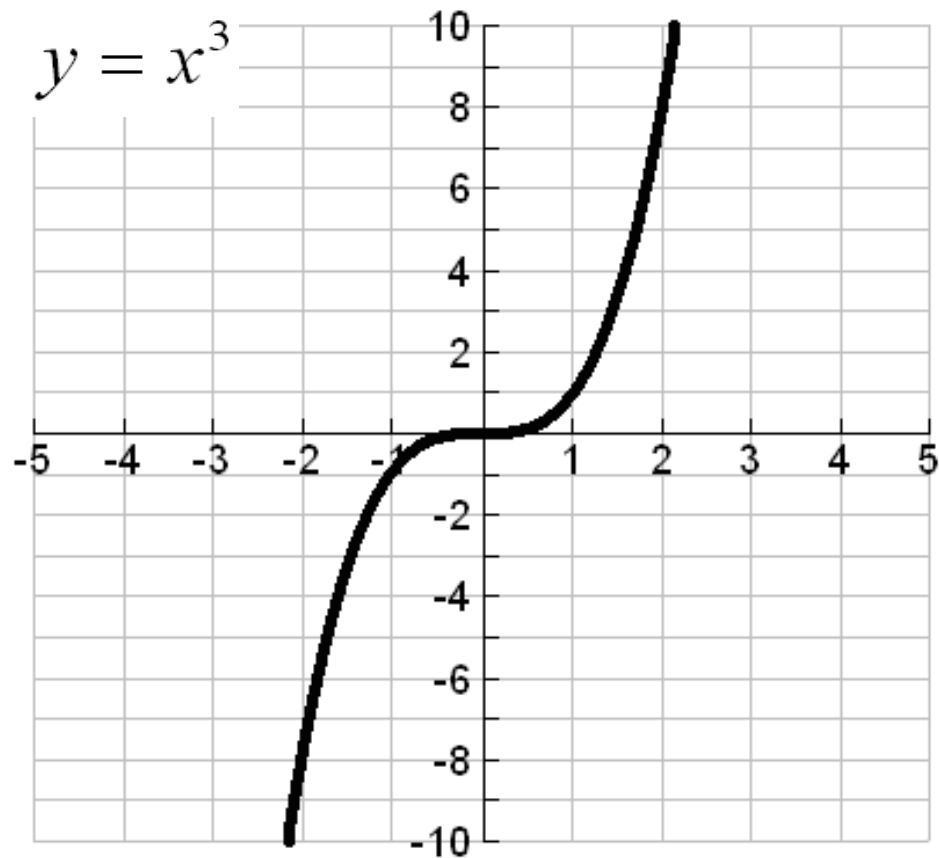
Graphically

x	f(x)
-3	9
-2	4
-1	1
1	1
2	4
3	9

Numerically

Algebraically

$$f(-x) = f(x)$$



x	f(x)
-3	-27
-2	-8
-1	-1
1	1
2	8
3	27

Algebraic

$$f(-x) = -f(x)$$



# Odd/Even/Neither

## Symmetry (card title)

Odd  $f(-x) = -f(x)$

symmetry with respect to the origin

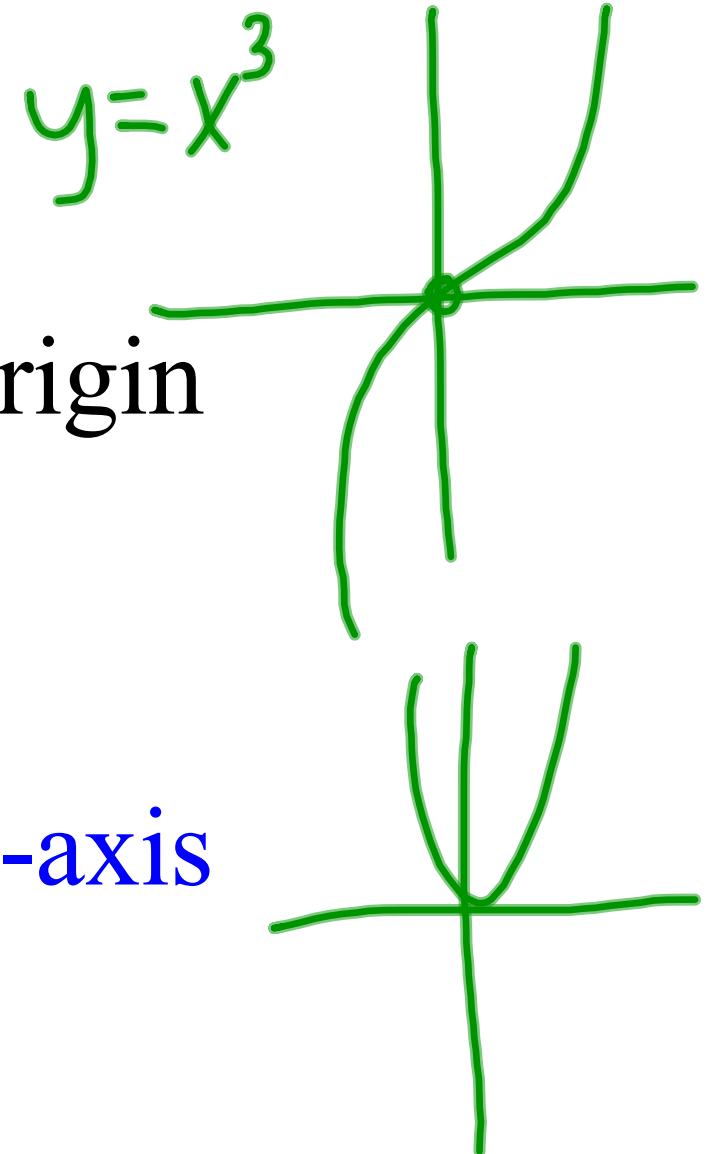
$f(x) = x^n$   $n$  is odd  
then  $f(x)$  is odd

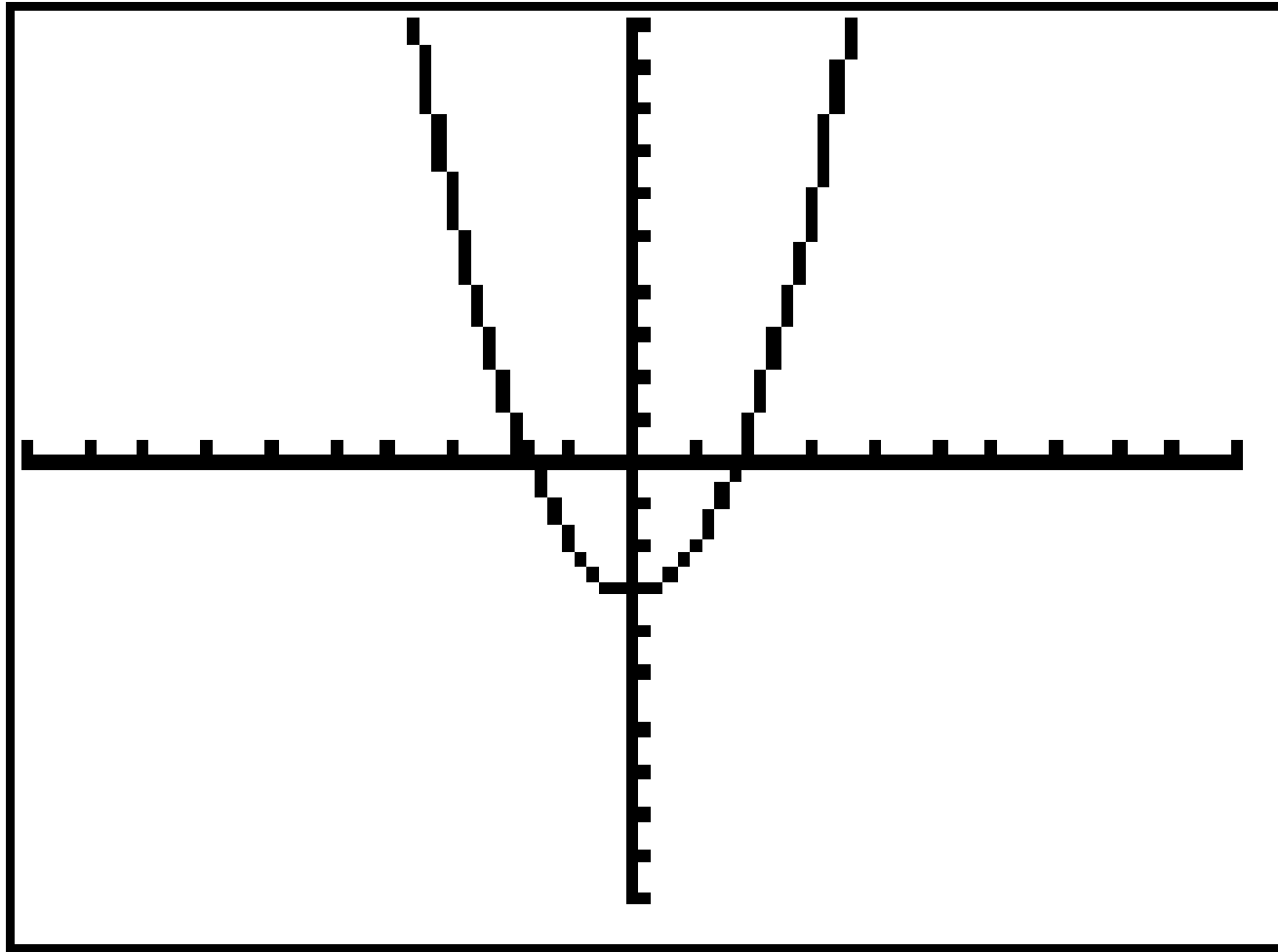
Even  $f(-x) = f(x)$

symmetry with respect to the y-axis

$f(x) = x^n$   $n$  is even  
then  $f(x)$  is even

Neither





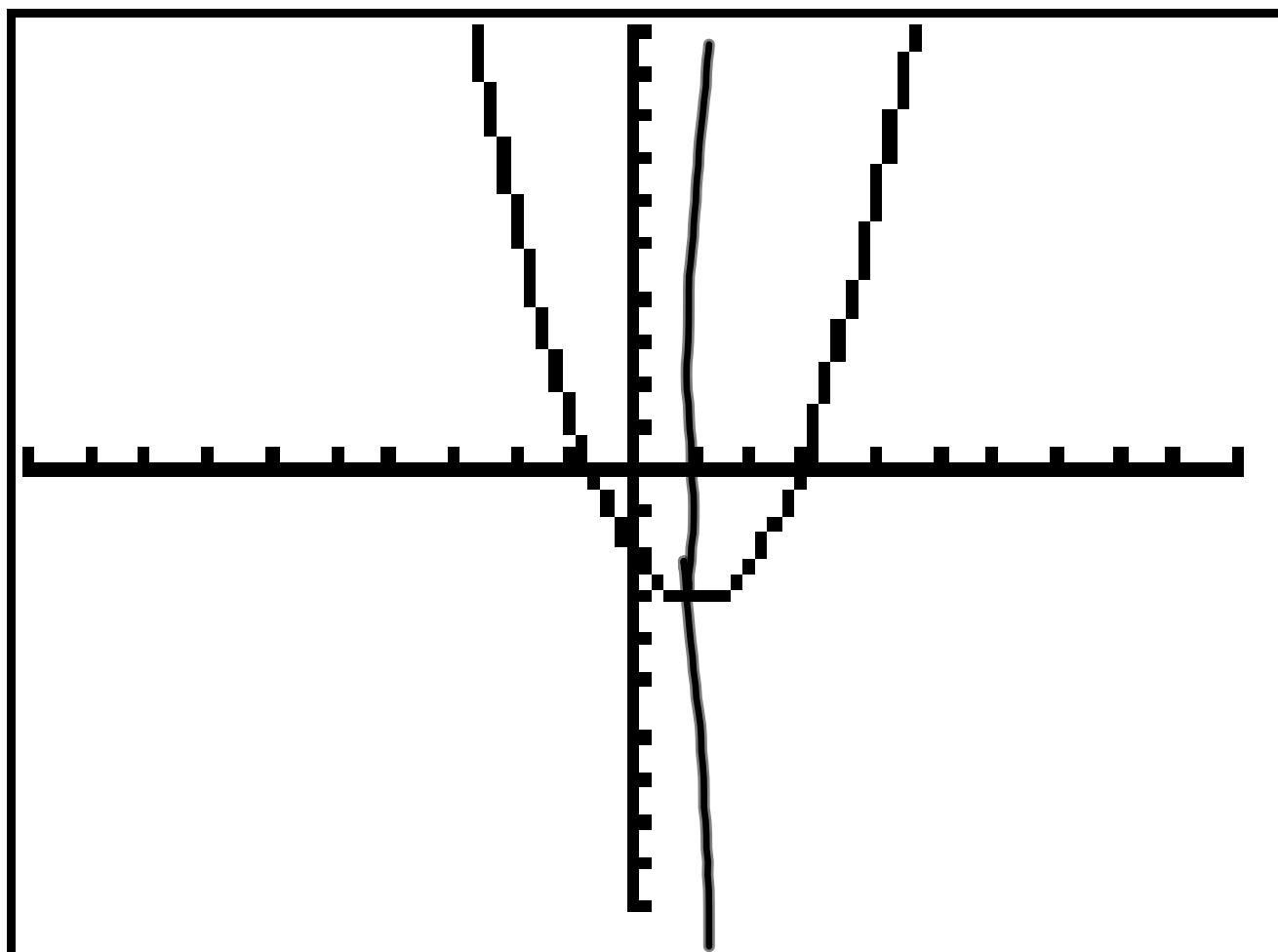
$$f(x) = x^2 - 3$$

$$f(-x) = f(x)$$

$$(-x)^2 - 3$$

$$x^2 - 3$$

EVEN



$$f(x) = x^2 - 2x - 2$$

$$f(-x) = f(x)$$

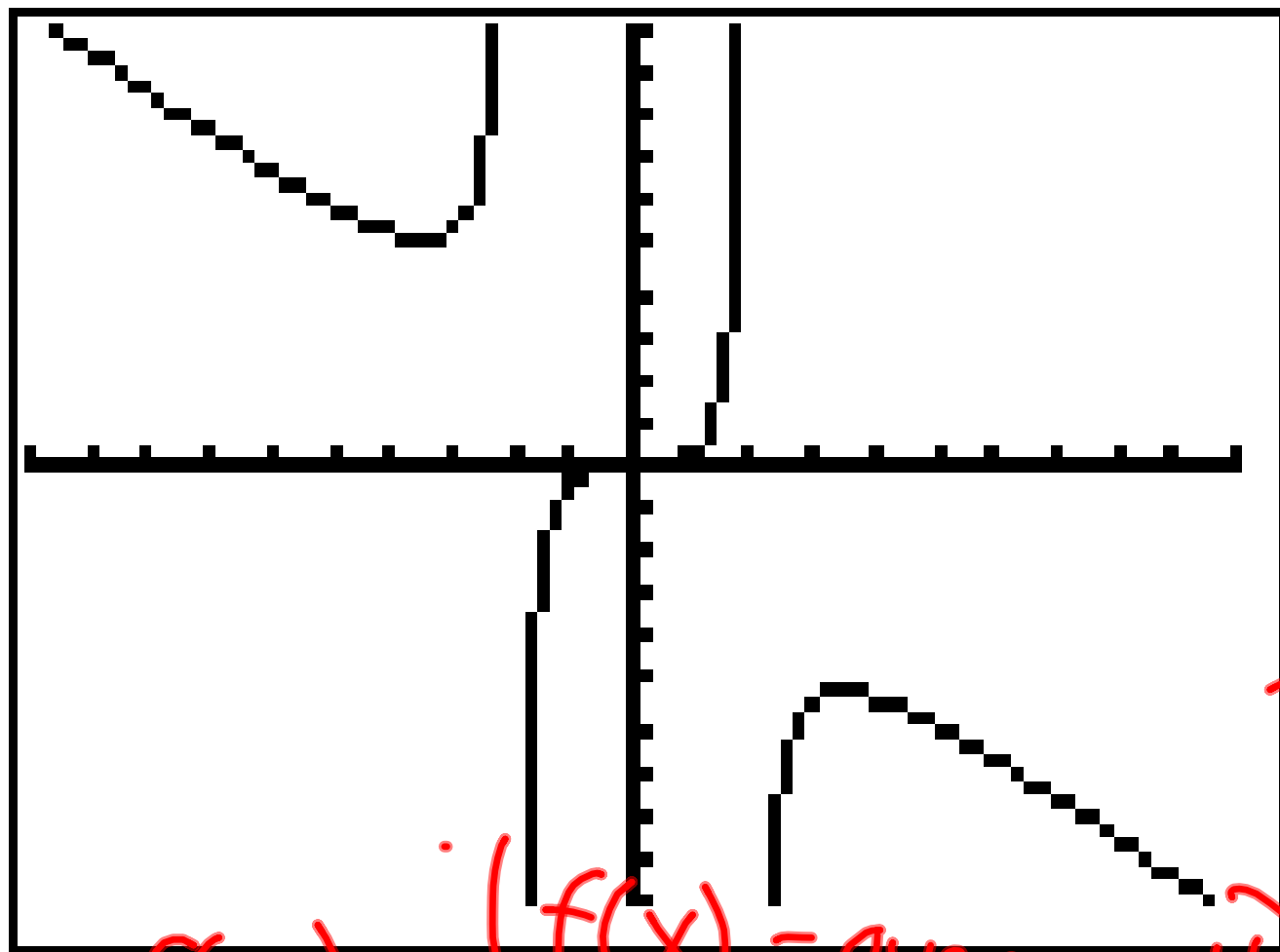
$$(-x)^2 - 2(-x) - 2$$

$$x^2 + 2x - 2$$

$$f(-x) = -f(x)$$

$$-(x^2 - 2x - 2)$$

$$-x^2 + 2x + 2$$



$$f(x) = \frac{x^3}{4 - x^2}$$

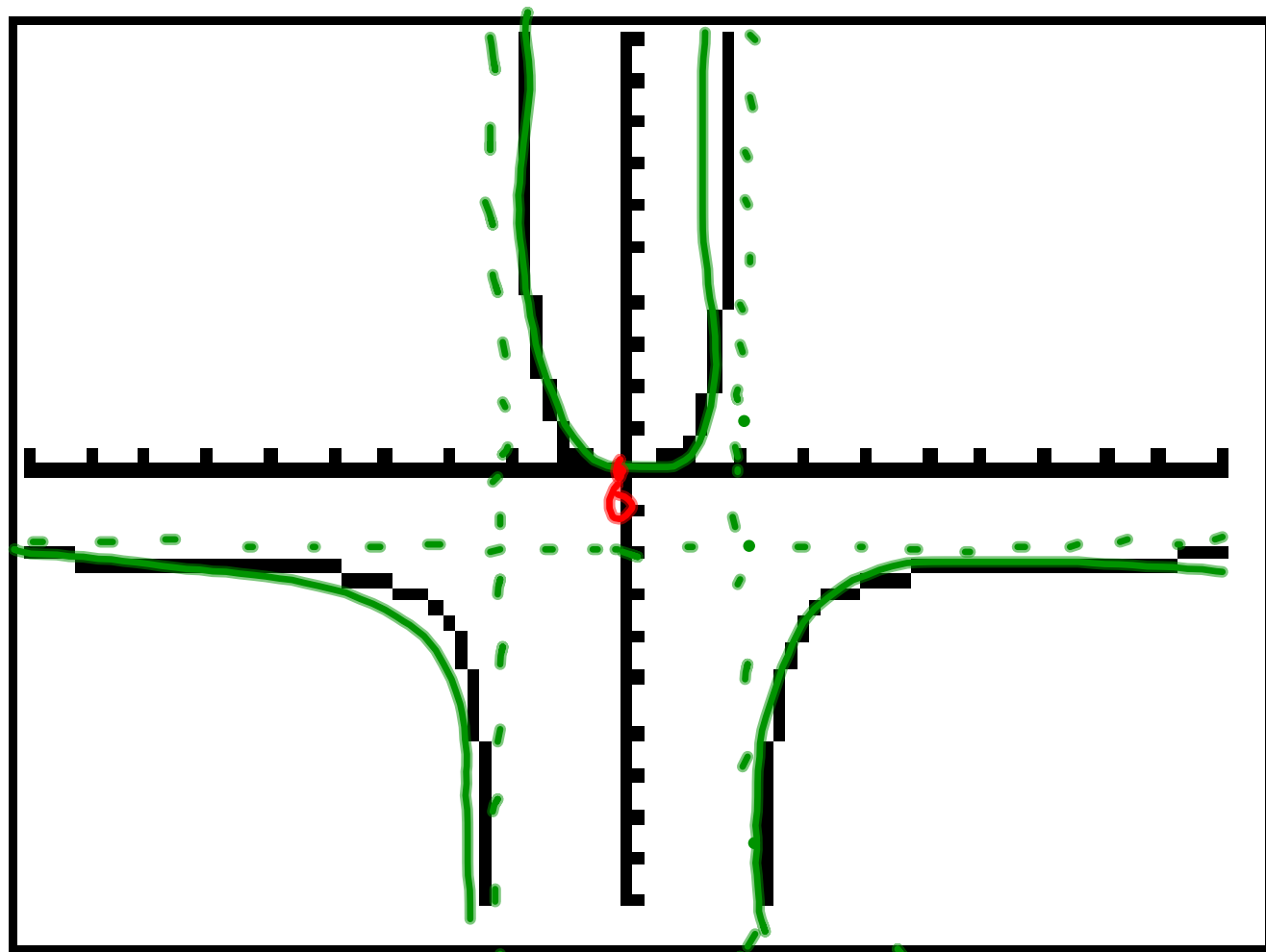
$$f(-x) = -f(x)$$

$$f(-x) = \frac{(-x)^3}{4 - (-x)^2} = \frac{-x^3}{4 - x^2}$$

$$f(-x) = \begin{cases} f(x) = \text{even} \\ -f(x) = \text{odd} \end{cases}$$

$$f(-x) = \left( \frac{-x^3}{4 - x^2} \right) = -\frac{x^3}{4 - x^2}$$

Odd



$$f(x) = \frac{2x^2}{4 - x^2}$$

Domain:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$   
 Range:  $(-\infty, -2) \cup [0, \infty)$

V.A.:  $x = -2$   $x = 2$   
 H.A.:  $y = -2/1 = -2$

Increasing:  $[0, 2) \cup (2, \infty)$

Decreasing:  $(-\infty, -2) \cup (-2, 0]$

Extrema:  $\text{min } y = 0$  (local)

Odd/Even/Neither:

$$f(-x) = \frac{2(-x)^2}{4 - (-x)^2} = \frac{2x^2}{4 - x^2} \text{ EVEN}$$



⑫  $\frac{1}{x} + \frac{5}{x-3}$   $0, 3$   
 $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

⑩  $\frac{5}{x-3}$   $3$   $(-\infty, 3) \cup (3, \infty)$

⑭  $\frac{\sqrt{4-x^2}}{x-3}$   $3, 2, -2$   
 $[-2, 2]$   $x \leq 2$   
 $x \geq -2$

