1.2 & 8.1 Exponents

$\ a^b \ = \ \text{Power}$

- **base**
- **Exponent**

$b$ is called the **Exponent** and it represents the number of times $a$, the **base** is used as a factor.

Powers represent repeated multiplication.

$$4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$
<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Words</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^1$</td>
<td>ten to the first power</td>
<td>10</td>
</tr>
<tr>
<td>$4^2$</td>
<td>four squared</td>
<td>$4 \cdot 4 = 16$</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$x$ to the $n^{th}$ power</td>
<td>$x \cdot x \cdot x \cdot \ldots \cdot x$ for $n$ times</td>
</tr>
</tbody>
</table>
Evaluating Exponents

What does it mean to evaluate? 

Evaluate $x^3$ for $x = 5$

$5^3 = 5 \cdot 5 \cdot 5 = 125$

Remember when you have parenthesis you must evaluate parenthesis FIRST:

Evaluate $2x^3$ and $(2x)^3$ for $x = 4$

$2 \cdot 4^3 = 2 \cdot 4 \cdot 4 \cdot 4 = 128$

$(2 \cdot 4)^3 = 8^3 = 8 \cdot 8 \cdot 8 = 512$
1. Write out the multiplication for $6^3 \cdot 6^5$

$$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$$

2. How many times did you write down 6? (How many factors of 6 are there?)

3. What do you notice about the factors and the exponents?

if you add 3+5 = 8

exponents
4. Write out the multiplication for \((6^2)^4\):

\((6 \cdot 6)(6 \cdot 6)(6 \cdot 6)(6 \cdot 6)\)

5. How many factors of six did you write? 8

6. What do you notice about the factors and the exponents? If you multiply the exponents \(2 \cdot 4 = 6\).
MULTIPLICATION PROPERTIES OF EXPONENTS

PRODUCT OF POWER PROPERTY
To multiply powers having the same base, **add** the exponents.

\[ a^m \cdot a^n = a^{m+n} \]

Example: \( 3^2 \cdot 3^7 = 3^{2+7} = 3^9 = 19683 \)

POWER OF A POWER PROPERTY
To find a power of a power, **multiply** the exponents.

\[ (a^m)^n = a^{m\cdot n} \]

Example: \( (5^3)^7 = 5^{3\cdot 7} = 5^{21} \)

POWER OF A PRODUCT PROPERTY
To find a power of a product, find the power of each factor and multiply

\[ (a \cdot b)^m = a^m \cdot b^m \]

Example: \( (2 \cdot 3)^6 = 2^6 \cdot 3^6 \)
*The power of a product property only works for PRODUCTS it does not work when addition is the operation used.

\[(a + 1)^3 = (a + 1)(a + 1)(a + 1)\]

\[(a + 1)^3 \neq a^3 + 1^3\]
Examples: Simplify the following:

<table>
<thead>
<tr>
<th>$x^2 \cdot x^3 \cdot x^4$</th>
<th>$(-2)(-2)^4$</th>
<th>$(y^2)^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^9$</td>
<td>$(-2)^8$</td>
<td>$2 \cdot 5 \cdot 10$</td>
</tr>
<tr>
<td>$2 \cdot 5 \cdot 10$</td>
<td>$y^{10}$</td>
<td></td>
</tr>
</tbody>
</table>
### Examples: Simplify the following:

- \((-3)^3\)^2
- \((4yz)^3\)
- \((4x^2y)^3 \cdot x^5\)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3)^3)^2 \rightarrow ((-27)^2) \rightarrow ((-3)^6)</td>
<td>((-27)^2) \rightarrow ((-3)^6)</td>
</tr>
<tr>
<td>((4yz)^3) \rightarrow (4^3 \cdot y^3 \cdot z^3) \rightarrow (64y^3z^3)</td>
<td>(64y^3z^3)</td>
</tr>
<tr>
<td>((4x^2y)^3 \cdot x^5) \rightarrow (4^3 \cdot x^6 \cdot y^3 \cdot x^5) \rightarrow (64x^{11}y^3)</td>
<td>(64x^{11}y^3)</td>
</tr>
</tbody>
</table>
\[ 1^6 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \]

\[ 10^3 = 10 \cdot 10 \cdot 10 = 1000 \]

\[ 10^{12} = 1,000,000,000,000,000 \]