1-4 Rational Exponents

Rational exponent Calc task

1, 4, 9, 16, 25, 36

$a^{\left(\frac{1}{2}\right)} = \_\_\_\_\_$

1, 8, 27, 64, 125, 216

$a^{\left(\frac{1}{3}\right)} = \_\_\_\_\_$

1, 8, 27, 64, 125, 216

$a^{\left(\frac{2}{3}\right)} = \_\_\_\_\_$

1, 16, 81, 256, 625, 1296

$a^{\left(\frac{3}{4}\right)} = \_\_\_\_\_$

$a^{\left(\frac{m}{n}\right)} = \_\_\_\_\_$

Fractional exponent

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

n is an integer bigger then or equal to 2

$a^{\frac{1}{3}} = \sqrt[3]{a}$

Write each of the following as a radical and simplify, if possible.

$9^{\frac{1}{2}}$ ← index

$\sqrt{9} = 3$

$(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64} = -4$

$100^{\frac{1}{2}}$

$\sqrt{100} = 10$

$-100^{\frac{1}{2}}$

$-\sqrt{100} = -10$

$z^{\frac{1}{2}}$

$\sqrt{z}$

You try

$25^{\frac{1}{2}}$

$(-27)^{\frac{1}{3}}$

$-64^{\frac{1}{2}}$

$b^{\frac{1}{2}}$

Rewrite in exponent form

$\sqrt[7]{x} = x^{\frac{1}{7}}$ $\qquad \sqrt[4]{b} = b^{\frac{1}{4}}$

You try

$\sqrt[12]{r}$ $\qquad \sqrt[5]{d}$

power inside radical

index

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Radicand

a is real, m/n is a rational number in lowest terms with n bigger or equal to 2

Write each of the following as a radical and simplify, if possible.

$25^{\frac{3}{2}}$ $\qquad \sqrt{25^3} = 125$ $\qquad 25 \cdot 25 \cdot 25$ $\qquad \sqrt{25}^3$

$64^{\frac{2}{3}}$ $\qquad = 16$ $\qquad \sqrt[3]{64^2}$ $\qquad 64 \cdot 64$

$-9^{\frac{5}{2}}$

$(-8)^{\frac{4}{3}}$ $\qquad -81^{\frac{3}{2}}$

You try

$27^{\frac{2}{3}}$ $16^{\frac{3}{2}}$

$-25^{\frac{5}{2}}$ $-16^{\frac{3}{4}}$

Rewrite in exponent form

$\sqrt[3]{x^2}$ $x^{\frac{2}{3}}$ $\left(\sqrt[4]{r}\right)^2$ $r^{\frac{2}{4}}$ $r^{\frac{1}{2}}$

You try

$\sqrt[8]{a^3}$ $a^{\frac{3}{8}}$ $\left(\sqrt[3]{h}\right)^9$ $h^{\frac{9}{3}}=h^3$

$a^{-\frac{m}{n}}=\frac{1}{a^{\frac{m}{n}}}$ and $\frac{1}{a^{-\frac{m}{n}}}=a^{\frac{m}{n}}$

$\frac{m}{n}$ is a rational number, and a is a nonzero real number

Write each of the following as a radical and simplify, if possible.

$x^{-\frac{1}{3}}$ $\quad \frac{1}{x^{1/3}}$

$36^{-\frac{1}{2}}$ $\quad \frac{1}{36^{1/2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

Just a reminder.

Exponent Rules

$a^0 = 1 \quad \text{if } a \neq 0$

$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n \quad \text{if } a \neq 0$

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } a \neq 0$

$(a^m)^n = a^{m \cdot n}$

$(a \cdot b)^n = a^n \cdot b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{if } b \neq 0$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \text{if } a \neq 0,\ b \neq 0$

After you simplify you should have:

- Only positive exponents.
- Each base only occurring once.
- Have no parentheses in the expression.
- No powers written to powers.

Simplify using properties of exponents. Leave answers with rational exponents

$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{3}{6}+\frac{2}{6}} = x^{5/6}$

$\frac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}} \quad x^{\frac{1}{3}-\frac{5}{3}} = x^{-\frac{4}{3}} = \frac{1}{x^{4/3}}$

You Try

$r^{\frac{3}{4}} \cdot r^{\frac{1}{6}}$ $r^{3\cdot\frac{3}{4}+\frac{1}{6}\cdot 2} = r^{\frac{9}{12}+\frac{2}{12}} = r^{\frac{11}{12}}$

$\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{5}}}$ $x^{5\cdot\frac{2}{5}-\frac{1}{5}\cdot 3} = x^{\frac{10}{15}-\frac{3}{15}} = x^{\frac{7}{15}}$

Simplify each of the following:

$\left(x^{\frac{2}{5}}\right)^{\frac{5}{4}} = x^{\frac{10}{20}} = x^{\frac{1}{2}}$

$\left(x^{\frac{1}{2}} \cdot y^{\frac{2}{3}}\right)^{\frac{3}{2}}$ $x^{\frac{3}{4}} y^{\frac{6}{6}} = x^{\frac{3}{4}} y$

You Try

$\left(100^{\frac{3}{8}}\right)^{\frac{4}{3}}$

$\left(a^{\frac{3}{2}} \cdot b^{\frac{5}{4}}\right)^{\frac{2}{3}}$

Simplify the following:

$\left(16x^{-4}y^{6}\right)^{-\frac{3}{2}}$

$16^{-\frac{3}{2}} x^{\frac{12}{2}} y^{-\frac{18}{2}}$

$\dfrac{x^{\frac{12}{2}}}{16^{\frac{3}{2}} y^{\frac{18}{2}}}$

$\dfrac{x^{6}}{\sqrt{16^{3}}} = \dfrac{x^{6}}{64y^{9}}$

$16 \cdot 16 \cdot 16$

Use rational exponents to simplify the radicals.

$$\frac{\sqrt{x}}{\sqrt[3]{x^2}}$$

$$\sqrt{\sqrt[3]{z}}$$