

## Radicals

Definition

 $n$ th root

$$\sqrt[n]{b} = a \text{ means } b = a^n$$

- if  $n \geq 2$  and even then  $a$  and  $b$  must be greater than or equal to 0.
- if  $n \geq 3$  and odd, then  $a$  and  $b$  can be any real number.

In  $\sqrt[n]{b}$ :The symbol  $\sqrt{\quad}$  is called the radical $n$  is called the index $b$  is called the radicand

if there is no index, it is 2

Evaluate

$$4 + 7 - 7$$

$$\sqrt{9} = 3$$

$$\sqrt{49} = 7$$

$$\sqrt[4]{16} = 2$$

$$\sqrt{3^2} = 3$$

$$\sqrt{7^2}$$

$$\sqrt[4]{2^4} = 2$$

$$\begin{array}{l} \textcircled{3} \sqrt{64} \\ \quad \swarrow \quad \searrow \\ \textcircled{2} \quad 32 \\ \quad \swarrow \quad \searrow \\ \quad 4 \quad 8 \\ \quad \swarrow \quad \searrow \\ \quad 2 \quad 4 \\ \quad \swarrow \quad \searrow \\ \quad 2 \quad 2 \quad 2 \quad 2 \\ \quad \swarrow \quad \searrow \\ \quad 2 \cdot 2 = 4 \end{array}$$

$\nearrow \sqrt[3]{-8} = -2$     $\sqrt[4]{81}$

odd

You try

$$\sqrt{121}$$

$$\sqrt[3]{125}$$

$$\sqrt[3]{-216}$$

$$\sqrt[5]{32}$$

Simplifying

If  $n \geq 2$  is a positive integer and  $a$  is a real number, then

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

Reduce

$$\sqrt{x^2} = |x| \quad \sqrt[5]{x^5} = x$$

You try

$$\sqrt[3]{x^3} \times$$

$$\sqrt[6]{z^6} \quad |z|$$

Simplify

$$\sqrt{18}$$

Simplify

(remember  $\sqrt{x^2} = |x|$ )

$$5\sqrt[3]{24} = 5\sqrt[3]{2^3 \cdot 3}$$

$$5 \cdot 2\sqrt[3]{3}$$

$$10\sqrt[3]{3}$$

$$\sqrt[4]{20}$$

$$\sqrt{128x^2}$$

$$8|x|\sqrt{2}$$

$$\sqrt{8^2 \cdot 2 \cdot x^2}$$

$$\sqrt{8^2} \cdot \sqrt{2} \cdot \sqrt{x^2}$$

$$8|x|\sqrt{2}$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Simplify

$$\sqrt{12p^2q}$$

$1p \sqrt{12q}$   
 $\begin{array}{c} 4 \quad 3 \\ \swarrow \quad \searrow \\ 2 \quad \sqrt{3q} \end{array}$   
 $2|p|\sqrt{3q}$

Remember that

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

$$\sqrt{x^2} = |x| \quad \sqrt[3]{x^3} = x \quad \sqrt[4]{x^4} = |x| \quad \text{and so on}$$

Even ROOTS =  $| |$   
 ODD ROOTS =  $( )$

Reduce

$$\sqrt{x^6} = \sqrt{\underbrace{x \cdot x}_{x} \cdot \underbrace{x \cdot x}_{x} \cdot \underbrace{x \cdot x}_{x}} = |x^3|$$

$$= |x^3|$$

$$\sqrt[3]{x^{12}} = x^4$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Reduce

$$\sqrt{20x^{10}}$$

.

You try

$$\sqrt{75a^6}$$

Simplify

$$\sqrt{80a^3}$$

$$\sqrt[3]{27m^4n^{14}}$$



You Try

$$\sqrt[3]{128x^6y^{10}}$$

A tree diagram showing the prime factorization of the radicand:
 

- 128 is factored into 64 and 2.
- 64 is factored into 8 and 8.
- 8 is factored into 2 and 4.
- 4 is factored into 2 and 2.
- The final factors are 2, 2, 2, 2, 2, 2, 2, and 2.

 The 2s are arranged in two columns of four. The first 2 in the first column is circled in green. The 2s in the second column are crossed out with green 'x' marks.

$$\sqrt[4]{16a^5b^{11}}$$

$$4x^2y^3\sqrt[3]{2y}$$

$$2|a|b^2\sqrt[4]{ab^3}$$