Radicals

Definition
*n*th root

$$\sqrt[n]{b} = a \quad \text{means} \quad b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{}$ is called the radical

n is called the index

b is called the radicand

if there is no index, it is 2

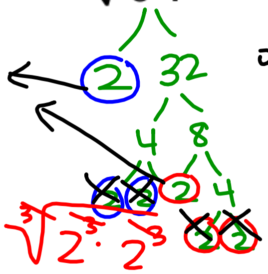
Evaluate $4 + 7 - 7$

$\sqrt{9} = 3$ $\quad \sqrt{49} = 7$ $\quad \sqrt[4]{16} = 2$

$\sqrt{3^2} = 3$ $\quad \sqrt{7^2}$ $\quad \sqrt[4]{2^4} = 2$

$\sqrt[3]{64}$ $\quad \sqrt[3]{-8} = -2$ $\quad \sqrt[4]{81}$

ODD



$\sqrt[3]{2^3 \cdot 2^3}$

$2 \cdot 2 = 4$

You try

$\sqrt{121}$ $\sqrt[3]{125}$

$\sqrt[3]{-216}$ $\sqrt[5]{32}$

Simplifying

If n≥2 is a positive integer and a is a real number, then

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

Reduce

$$\sqrt{x^2} = |x| \qquad \sqrt[5]{x^5} = x$$

You try

$$\sqrt[3]{x^3}\ x \qquad \sqrt[6]{z^6}\ |z|$$

Simplify

$$\sqrt{18}$$

Simplify

(remember $\sqrt{x^2} = |x|$)

$$5\sqrt[3]{24} = 5\sqrt[3]{2^3 \cdot 3}$$

$$5 \cdot 2\sqrt[3]{3}$$

$$10\sqrt[3]{3}$$

2 12

$\cancel{2}$ 6

$\cancel{2}$ 3

$10\sqrt[3]{3}$

$$\sqrt{128x^2}$$

64 2 x x

8 $\cancel{8}$

$8|x|\sqrt{2}$

$$\sqrt{8^2 \cdot 2 \cdot x^2}$$

$$\sqrt{8^2} \cdot \sqrt{2} \cdot \sqrt{x^2}$$

$$8|x|\sqrt{2}$$

$$\sqrt[4]{20}$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Simplify

$$\sqrt{12p^2q}$$

$$|p|\sqrt{12q}$$

$$4 \quad 3$$

$$2 \quad \cancel{2}$$

$$2|p|\sqrt{3q}$$

Remember that

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

$\sqrt{x^2} = |x|$ $\quad \sqrt[3]{x^3} = x$ $\quad \sqrt[4]{x^4} = |x|$ and so on

Even ROOTS = | |

ODD ROOTS = $\cancel{(\ |\ )}$

Reduce

$$\sqrt{x^6} = \sqrt{\underbrace{x \cdot x}_{x} \cdot \underbrace{x \cdot x}_{x} \cdot \underbrace{x \cdot x}_{x}} = |x^3|$$

$$= |x^3|$$

$$\sqrt[3]{x^{12}} = x^4$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Reduce

$$\sqrt{20x^{10}}$$

You try

$$\sqrt{75a^6}$$

Simplify

$$\sqrt{80a^3}$$

$$\sqrt[3]{27m^4n^{14}}$$

You Try

$$\sqrt[3]{128x^6y^{10}}$$

64 2

8 8

2 4 2 4

$\cancel{2}$ $\cancel{2}$ $\cancel{2}$ $\cancel{2}$

$4x^2y^3\sqrt[3]{2y}$

$$\sqrt[4]{16a^5b^{11}}$$

$2|a|b^2\sqrt[4]{ab^3}$